Techniques of Differentiation

I. Positive Integer Powers, Multiples, Sums, and Differences

A.) Th: If $f(x)$ is a constant, $f(x)=k \Rightarrow f'(x)=0$

B.) Th: **The Power Rule:** If n is a positive integer and,

$$f(x)=x^n \Rightarrow f'(x)=nx^{n-1}$$

C.) Th: **The Constant Multiple Rule:** If k is a real number and f is differentiable at x, then

$$\frac{d}{dx}\left(kf(x)\right) \Rightarrow kf'(x)=k\frac{df}{dx}\left(f(x)\right)$$

D.) Th: **The Sum/Difference Rule:** If f and g are differentiable functions of x at x, then their sum and their difference are differentiable at ever point where f and g are differentiable

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) \Rightarrow \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$$

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$$

II. Examples

A.) Find the following derivatives:

1.) $f(x) = 2$

$f'(x) = 0$

2.) $f(x) = x^3$

$f'(x) = 3x^2$

3.) $f(x) = \frac{x^4}{3}$

$f'(x) = \frac{4}{3}x^3$

4.) $f(x) = 3x^3 + 4x^2 - 2x + 1$

$f'(x) = 9x^2 + 8x - 2$

III. Derivative Notation for Functions of *x*

A.) Given u as a function of x, the derivative of u is written as follows:

$$\frac{d}{dx}[u] = \frac{du}{dx}$$

IV. Product Rule $\frac{d}{dx}[uv]$

A.) Th: $\frac{d}{dx}[uv] = v\frac{du}{dx} + u\frac{dv}{dx}$

B.) Use the product rule to find the derivatives of the following functions:

1.) $f(x) = 2x^2(3x+1)$

$$f'(x) = 4x(3x+1) + 3\left(2x^2\right)$$

$$f'(x) = 12x^2 + 4x + 6x^2$$

$$f'(x) = 18x^2 + 4x$$

2.) $g(x) = \left(x^2+1\right)(x^3+3)$

$$g'(x) = 2x(x^3+3) + 3x^2\left(x^2+1\right)$$

$$g'(x) = 2x^4 + 6x + 3x^4 + 3x^2$$

$$g'(x) = 5x^4 + 3x^2 + 6x$$

C.) Let $y = uv$ be the product of the functions u and v. Find $y'(2)$ if $u(2) = 3$, $u'(2) = -4$, $v(2) = 1$, and $v'(2) = 2$.

$$y' = vu' + uv'$$

$$y'(2) = v(2)u'(2) + u(2)v'(2)$$

$$y'(2) = 1\left(-4\right) + 3\left(2\right)$$

$$y'(2) = 2$$

V. Quotient Rule $\frac{d}{dx}\left[\frac{u}{v}\right]$

A.) Th: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

B.) Use the quotient rule to find the derivatives of the following functions:

1.) $f(x) = \frac{2x^2}{3x+1}$

$$f'(x) = \frac{4x(3x+1) - 3\left(2x^2\right)}{\left(3x+1\right)^2}$$

$$f'(x) = \frac{6x^2 + 4x}{\left(3x+1\right)^2}$$

2.) $g(x) = \frac{\left(x^2 - 1\right)}{x}$

$$g'(x) = \frac{2x(x) - 1(x^2 - 1)}{x^2}$$

$$g'(x) = \frac{x^2 + 1}{x^2}$$

VI. Negative Exponent Thm:

A.) For any integer $n \neq 0$,

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

B.) Find the following derivative using both the quotient rule and the negative exponent theorem.

$$f(x) = \frac{1}{x^2} \qquad f(x) = x^{-2}$$

$$f'(x) = \frac{0-2x}{\left(x^2\right)^2} = -\frac{2}{x^3} \qquad f'(x) = -2x^{-3}$$

VI. Tangent Lines

A.) Find the instantaneous rate of change of the following functions at the given *x* values. Then, give the equation of the tangent line at that point.

1.) $f(x)=x^2$, at $x=3$

$$f(3)=(3)^2=9$$

$$f'(x)=2x$$

$$f'(3)=6$$

$$y-9=6(x-3)$$

2.) $f(x)=\frac{5}{x}$, at $x=1$ $\quad f(1)=\frac{5}{1}=5$

$$f'(x)=\frac{0(x)-1(5)}{x^2}=-\frac{5}{x^2}$$

$$f'(1)=-\frac{5}{(1)^2}=-5$$

$$y-5=-5(x-1)$$

VII. Derivatives of Trig Functions

A.) $\frac{d}{dx}[\sin x] = \cos x$

B.) $\frac{d}{dx}[\cos x] = -\sin x$

C.) $\frac{d}{dx}[\tan x] = \sec^2 x$

D.) $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

VIII. Examples

A.) Find the following derivatives:

1.) $y = 2x \tan x$ $\qquad$ $y' = 2\tan x + 2x\sec^2 x$

2.) $y = \dfrac{\sin x}{1+\cos x}$ $\qquad$ $y' = \dfrac{1}{1+\cos x}$