Derivation for Sum of a Finite Arithmetic Sequence

$$\sum_{k=1}^{n} a_k = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_1 + (n-1)d)$$

$$= a_n + (a_n - d) + (a_n - 2d) + \ldots + (a_n - (n-1)d)$$

Sum the preceding lines vertically:

$$2\sum_{k=1}^{n} a_k = (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \ldots + (a_1 + a_n)$$

$$2\sum_{k=1}^{n} a_k = n(a_1 + a_n)$$

$$\sum_{k=1}^{n} a_k = n\left(\frac{a_1 + a_n}{2}\right)$$

Derivation for Sum of a Finite Geometric Sequence

$$\sum_{k=1}^{n} g_k = g_1 + g_1 r + g_1 r^2 + \ldots + g_1 r^{n-1}$$

$$r\sum_{k=1}^{n} g_k = g_1 r + g_1 r^2 + \ldots + g_1 r^{n-1} + g_1 r^n$$

Subtract lower summation from upper summation:

$$(1-r)\sum_{k=1}^{n} g_k = g_1 - g_1 r^n$$

$$(1-r)\sum_{k=1}^{n} g_k = g_1(1 - r^n)$$

$$\sum_{k=1}^{n} g_k = \frac{g_1(1-r^n)}{(1-r)}$$