Notes 9.3 – Sequences

I. Sequences

A.) A progression of numbers in a pattern.

1.) FINITE – A set number of terms

$$2,\ 5,\ 8,\ 11, \ldots\ 242$$

2.) INFINITE – Continues forever

$$1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}, \ldots$$

B.) Notation - a_k where a is the sequence and k refers to the k^{th} term.

C.) Explicitly defined sequence– Allows us to substitute k into an equation to find the k^{th} term.

D.) Recursively defined sequence– Defines each term by using the previous term.

E.) Ex. 1 - Define the following sequence both explicitly and recursively.

$$4,\ 10,\ 16,\ 22, \ldots$$

Exp:

$$a_k = \text{first} + \text{diff}(k - 1)$$

$$a_k = 4 + 6(k - 1)$$
$$= -2 + 6k$$

Rec:

$$a_1 = 4$$
$$a_k = a_{k-1} + 6 \text{ for } k \geq 2$$

II. Types of Sequences

A.) Arithmetic Sequence – any sequence with a common difference between terms.

$$2,\ 5,\ 8,\ 11, \ldots$$

B.) Geometric Sequence - any sequence with a common ratio between terms.

$$1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}, \ldots$$

C.) Ex. 2- Define the following sequence both explicitly and recursively.

$$a = 84,\ 63,\ \frac{189}{4},\ \frac{567}{16}, \ldots$$

Exp.

$$a_k = \text{first} \cdot \left(\text{ratio}^{(k\text{-}1)}\right)$$

$$a_k = 84\left(\frac{3}{4}\right)^{k-1}$$

Rec.

$$a_1 = 84$$

$$a_k = \frac{3}{4}\left(a_{k-1}\right) \text{ for } k \geq 2$$

D.) Constructing a Sequence – Ex. 3 - The third and sixth terms of a sequence are -12 and 48 respectively.

Find an explicit formula for the sequence if it is

1.) arithmetic and 2.) geometric

1.) - $-12 = a_1 + 2d$

$48 = a_1 + 5d$

$a_1 = -12 - 2d$

$a_1 = 48 - 5d$

$-12 - 2d = 48 - 5d$

$d = 20$

$a_1 = -52$

$$a_k = -52 + 20(k-1)$$

2.)

Divide the two equations

$$\frac{48 = a_1 r^5}{-12 = a_1 r^2}$$

$-4 = r^3$

$r = \sqrt[3]{-4}$

$-12 = a_1\left(\sqrt[3]{-4}\right)^2$

$a_1 = -\frac{12}{\left(\sqrt[3]{-4}\right)^2}$

$a_k = -\frac{12}{\left(\sqrt[3]{-4}\right)^2}\left(\sqrt[3]{-4}\right)^{k-1}$

$$a_k = -12\left(\sqrt[3]{-4}\right)^{k-3}$$

III. Fibonacci Sequence

A.) Summation of successive terms. Can only be defined recursively

B.) Ex. 4– a_k = 1, 1, 2, 3, 5, 8, 13,…

$$a_1 = 1$$
$$a_2 = 1$$
$$a_k = a_{k-2} + a_{k-1} \text{ for } k \geq 3$$