Notes 9.2 – The Binomial Theorem

I. Alternate Notation

A.) Permutations – None

B.) Combinations - $_{n}C_{r} = \binom{n}{r}$

II. Pascal's Triangle

A.) A triangular array of the coefficients of the binomial coefficients of *a* and *b* in the expansion of $(a+b)^n$ where *n* = 0 is the top row.

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ . \ . \ . \ . \ . \ . \ . \ . \end{array}$$

B.) Recursion Formula for row *n* of Pascal's Triangle -

$$\binom{n}{0} \quad \binom{n}{1} \quad \cdots \quad \binom{n}{n-1} \quad \binom{n}{n}$$

C.) Ex. 1 - Determine the row 6 of Pascal's Triangle

$$\binom{6}{0} \quad \binom{6}{1} \quad \cdots \quad \binom{6}{5} \quad \binom{6}{6}$$

$$1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$$

III. The Binomial Theorem

A.) Thm.: for any positive integer *n*,

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \ldots \binom{n}{r}a^{n-r}b^r + \ldots \binom{n}{n}b^n$$

where $$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

B.) Ex. 2 - Expand $\left(\sqrt{2}x+3y^2\right)^5$

$$\left(\sqrt{2}x+3y^2\right)^5 = \binom{5}{0}\left(\sqrt{2}x\right)^5 + \binom{5}{1}\left(\sqrt{2}x\right)^4 3y^2 + \binom{5}{2}\left(\sqrt{2}x\right)^3\left(3y^2\right)^2 \ldots \binom{5}{5}\left(3y^2\right)^5$$

$$= 4\sqrt{2}x^5 + 60x^4y^2 + 180\sqrt{2}x^3y^4 + 540x^2y^6 + 405\sqrt{2}xy^8 + 243y^{10}$$

C.) Ex. 3 - Find the x^6y^3 term of $\left(2x+\sqrt{3}y\right)^9$

$$=\binom{9}{3}(2x)^6\left(\sqrt{3}y\right)^3=16128\sqrt{3}x^6y^3$$

D.) Ex. 4 - Find the sum of the coefficients of $\left(2x-3y\right)^{101}$

$$=(2-3)^{101}=-1$$

E.) Thm.: - The sum of the coefficients of $\left(ax\pm by\right)^n=\left(a\pm b\right)^n$

E.) Prove $\binom{n+1}{2}-\binom{n}{2}=n$ for all $n \geq 2$.

$$\frac{(n+1)!}{2!(n-1)!}-\frac{n!}{2!(n-2)!}=$$

$$\frac{(n+1)(n)(n-1)\ldots}{2(n-1)\ldots}-\frac{n(n-1)(n-2)\ldots}{2(n-2)\ldots}=$$

$$\frac{(n+1)(n)}{2}-\frac{n(n-1)}{2}=$$

$$\frac{n(n+1)-n(n-1)}{2}=$$

$$\frac{n^2+n-n^2+n}{2}=$$

$$\frac{2n}{2}=$$

$$n=n$$ ✓✓