Notes 9.1 – Basic Combinatorics

I. Types of Data

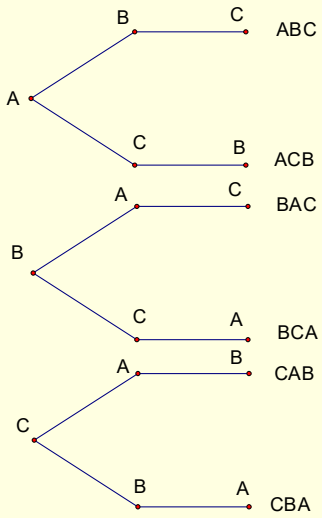
1.) Discrete Data – “Countable”; how many?

2.) Continuous Data – “uncountable”; measurements; you can always get a closer approximation.

II. The Multiplication Principle of Counting

Example 1: In how many ways can three students sit in three seats?

$$\underset{\text{Seat 1}}{3} \cdot \underset{\text{Seat 2}}{2} \cdot \underset{\text{Seat 3}}{1} = 6$$



How many different ways can 5 friends stand in a line?

$$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$$

III. Permutations

A.) *n* items arranged in an order.

$3! = 6$ permutations of 3-set.

n! distinguishable permutation of an *n*-set.

B.) Ex. 1- A typical NJ License plate consist of 3 letters followed by 3 digits. How many possible plates are there if repeats of digits and letters are allowed?

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$$

C.) Ex. 2- How many possible plates are there if repeats of digits and letters are not allowed?

$$26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11,232,000$$

D). How many 3-digit numbers can be selected in each of the following situations?

If you can use each digit more than once.

$$9 \cdot 10 \cdot 10 = \boxed{900}$$

If you cannot repeat digits.

$$9 \cdot 9 \cdot 8 = \boxed{648}$$

note: we started with the second digit, becuase it has less restrictions than the first, since 0 is allowed

E.) Ex. 3 - How many different 6 letter “words” can be formed by the word

1.) “VOLUME”?

$$6! = 720$$

2.) “DABBLE”?

$$\frac{6!}{2!} = 360$$

F.) Ex. 4 - How many different11 letter “words” can be formed by the word “MISSISSIPPI”?

$$\frac{11!}{4!4!2!} = 34650$$

F.) Distinguishable Permutations- If an n-set contains $n_1, n_2, \ldots n_k$ of a separate kind, then the number of permutations is

$$\frac{n!}{n_1!n_2!\ldots n_K!}$$

IV. Permutation Counting Formula

A.) The number of permutations of *n* objects taken *r* at a time is given by the formula

$$_nP_r = \frac{n!}{(n-r)!}$$

B.) Ex. 5- Provide an algebraic expression equivalent to

$$_nP_3 \qquad _nP_3 = \frac{n!}{(n-3)!}$$

$$_nP_3 = \frac{n(n-1)(n-2)(n-3)(n-4)\ldots}{(n-3)(n-4)\ldots}$$

$$_nP_3 = n(n-1)(n-2)$$

C). Evaluate each of the following without a calculator:

1. ${}_8P_4 = \frac{8!}{4!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!} = 8\cdot 7\cdot 6\cdot 5 = \boxed{1680}$

2. ${}_{11}P_4 = \frac{11!}{7!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7!}{7!} = 11\cdot 10\cdot 9\cdot 8 = \boxed{7920}$

3. ${}_nP_2 = n(n-1) = \boxed{n^2 - n}$

V. Combinations

A.) Use a COMBINATION anytime you are "selecting" or "choosing" an *r*-set of items from an *n*-set of items. NOTE: With combinations, order does not matter!!!

$$ {}_nC_r = \frac{n!}{r!(n-r)!} = \frac{{}_nP_r}{r!} $$

B.) Ex. 6 - How many different subcommittees of 4 people are possible if they are to be chosen from a group of 10 people?

$$_{10}C_4 = \frac{10!}{4!6!} = 210$$

BE CAREFUL!!! Elected officials – Permutations; Committees- Combinations

C.) Subsets of an *n*-set :

Ex. 7 - A pizzeria offers a choice of 10 toppings. How many possible pizzas are there if you order 4 toppings?

$$_{10}C_4 = \frac{10!}{4!6!} = 210$$

D.) Ex. 8 - How many total possible pizzas are there?

$$2^{10} = 1024$$