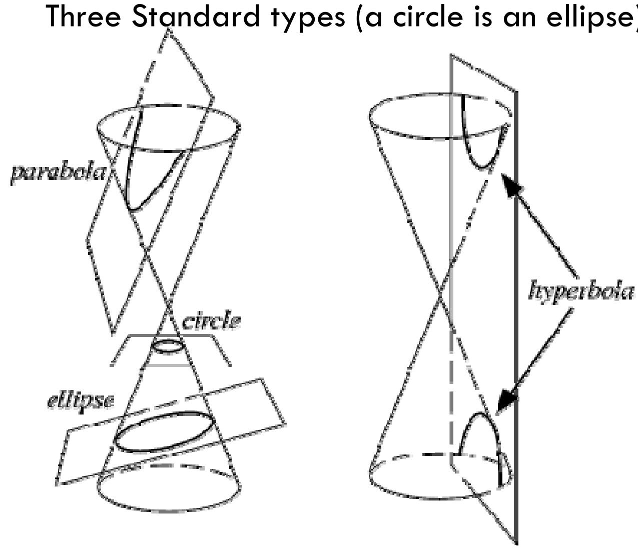
8.1: CONIC SECTIONS AND PARABOLAS

Conic Sections

Definition: A *conic section* is a curve obtained as the intersection of a cone and a plane

Three Standard types (a circle is an ellipse):



Degenerate conic sections

1. A point (through cones vertex)
2. A single line (plane tangent to a cone)
3. Intersecting lines (vertical plane through vertex point)

All conic sections can be defined algebraically as the graphs of *second degree(quadratic) equations in two variables.*

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Notes:

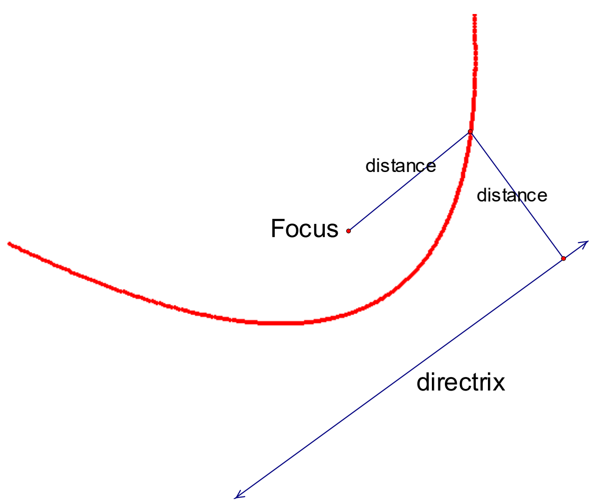
If both B and C are zero, it is an up or down parabola

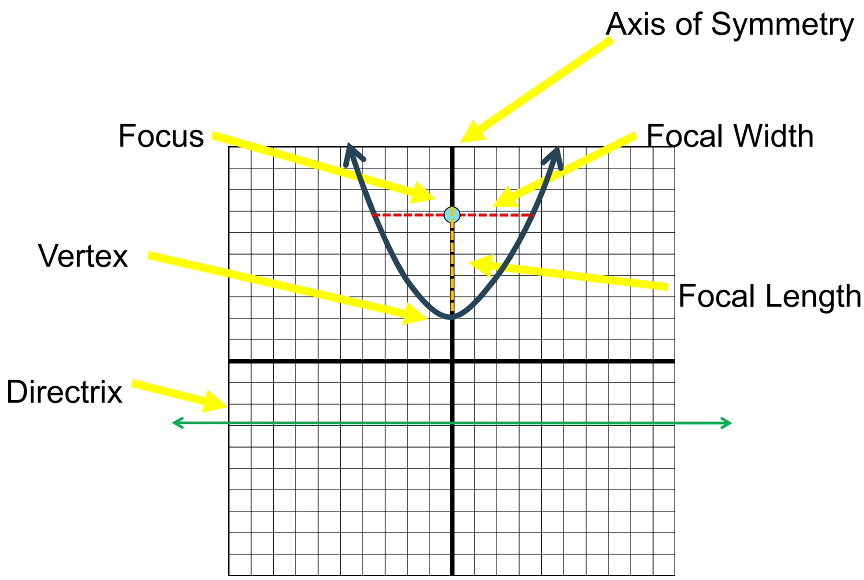
If both A and B are zero, it is a left or right parabola

Geometry of a Parabola

A *parabola* is the set of all points whose distance from a fixed line *D* equals the distance from a fixed point *F* not on the line.

1. The line ***d***
2. The point



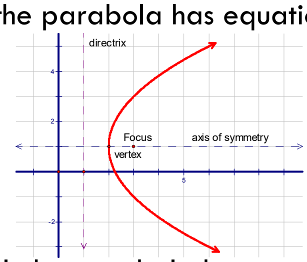


Standard forms of the equation of a parabola

Suppose that a parabola has vertex at *(h,k)* and the focal length (the directed distance from the vertex to the focus) is *p*.

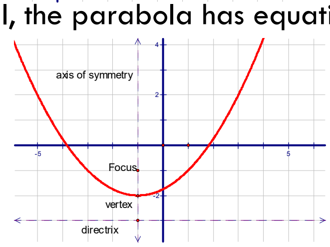
1. If the directrix is vertical, the parabola has equation

$$(y-k)^2 = 4p(x-h)$$



2. If the directrix is horizontal, the parabola has equation

$$(x-h)^2 = 4p(y-k)$$



Parabolas (Vertex = (0,0))

Standard Form	$x^2 = 4py$	$y^2 = 4px$
Focus	$(0, p)$	$(p, 0)$
Directrix	$y = -p$	$x = -p$
Axis of Symmetry	$y-$axis	$x-$axis
Focal Length	p	p
Focal Width	$\lvert 4p \rvert$	$\lvert 4p \rvert$

Parabolas (Vertex = (*h*, *k*))

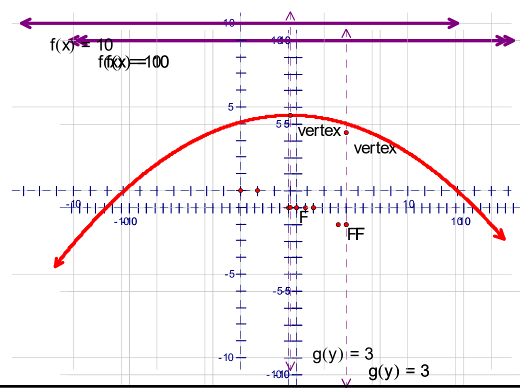
St. Fm.	$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Focus	$(h, k+p)$	$(h+p, k)$
Directrix	$y = k - p$	$x = h - p$
Ax. of Sym.	$x = h$	$y = k$
Fo. Lgth.	p	p
Fo. Wth.	$\lvert 4p \rvert$	$\lvert 4p \rvert$

Ex 1: Determine the equation of a parabola with focus *(3,-1)* and directrix *y=10*.

- Directrix is horizontal, so the axis is vertical, with equation *x=3*.
- The vertex is midway between the directrix and the focus:
- *p=-1-9/2=-11/2*

$$(x-3)^2 = 4\left(-\frac{11}{2}\right)\left(y-\frac{9}{2}\right)$$

$$\boxed{(x-3)^2 = -22\left(y-\frac{9}{2}\right)}$$



Ex 2: Determine the focus and the directrix of the parabola with the equation below, then sketch the parabola

$$y = 3x^2 - 6x$$

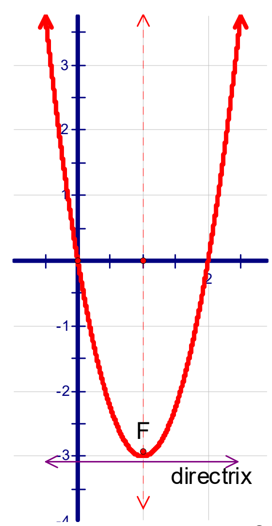
$$y = 3\left(x^2 - 2x\right)$$

$$y = 3\left(x^2 - 2x + 1\right) - 3$$

$$y + 3 = 3(x-1)^2$$

$$(x-1)^2 = \frac{1}{3}(y+3)$$

$$(x-1)^2 = 4\left(\frac{1}{12}\right)(y+3)$$



$$vertex:(1,-3),\ p = \tfrac{1}{12},\ focus:\left(1,-\tfrac{35}{12}\right), directrix: y = -\tfrac{37}{12}$$

Ex 3: Determine the focus and the directrix of the parabola with the equation below, then sketch.

$$y^2 + 2y = -x - 5$$

$$y^2 + 2y + 1 = -x - 4$$

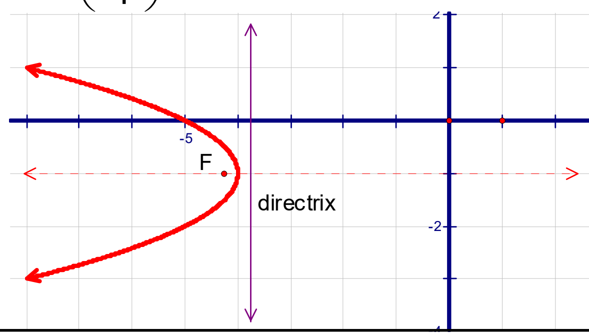
$$(y+1)^2 = -1(x+4)$$

$$(y+1)^2 = 4\left(\frac{-1}{4}\right)(x+4)$$

$vertex: (-4, -1),\ p = -\frac{1}{4},$

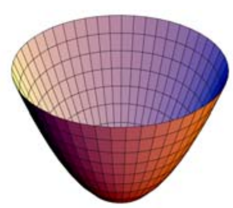
$focus: \left(-\frac{17}{4}, -1\right),$

$directrix: x = -\frac{15}{4}$



Application

When a parabola is rotated around its axis of symmetry, the three dimensional figure is called a *paraboloid of revolution.*



These shapes are used in headlights, satellite dishes, flashlights, microphones, and burning mirrors (thanks to Archimedes).

Ex 4: The headlight of a car is in the shape of a *paraboloid of revolution*. The radius of the headlight is 6 inches, and the distance from the vertex to the front is 4.5 inches.   How far from the vertex of the parabola should the light source be placed to maximize the amount of light emitted?

$$y^2 = 4px$$

$$(6)^2 = 4p(4.5)$$

$$p = 2$$

$$\boxed{2in.}$$