

7-6: Natural Logarithms

Algebra 2
Mr. Gallo

Natural Logarithmic Function

- The inverse of $y = e^x$ is the **Natural Logarithm**.

If $y = e^x$, then $x = \log_e y = \ln y$

The inverse is $y = \ln x$

$$y = e^x$$

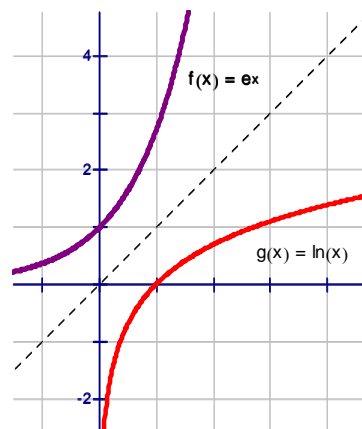
Domain: **All Reals**

Range: **$y > 0$**

$$y = \ln x$$

Domain: **$x > 0$**

Range: **All Reals**



Properties of Natural Logarithms

- All properties of logarithms apply

Product Property $\ln mn = \ln m + \ln n$

Quotient Property $\ln \frac{m}{n} = \ln m - \ln n$

Power Property $\ln m^n = n \ln m$

Complete Got It? #1 p.479

a. $\ln 175$ b. $\ln \frac{x}{4}$ c. $\ln 5x^3y^2$

Solving Natural Logarithmic Equations

What is the solution of the following logarithmic equation?

1). $\ln x = 5$ 2). $\ln(6x+1) = 3$ 3). $\ln(2x+1)^2 = 6$

$$e^5 = x$$

$$148.41 \approx x$$

$$e^3 = 6x+1$$

$$e^3 - 1 = 6x$$

$$\frac{e^3 - 1}{6} = x$$

$$3.181 \approx x$$

$$e^6 = (2x+1)^2$$

$$\sqrt{e^6} = \sqrt{(2x+1)^2}$$

$$\pm e^3 = 2x+1$$

$$-1 \pm e^3 = 2x$$

$$\frac{-1 \pm e^3}{2} = x$$

$$-10.54 \text{ or } 9.54 \approx x$$

Complete Got It? #2 p.479

a. 7.39 b. -4.13 or .769 c. 1.23

Solving Exponential Equations

- Take ln (natural log) of each side

$$1). e^{x+3} = 24$$

$$\ln e^{x+3} = \ln 24$$

$$x + 3 = \ln 24$$

$$x = \ln 24 - 3$$

$$x \approx .178$$

$$2). 5e^{-3x} = 45$$

$$e^{-3x} = 9$$

$$\ln e^{-3x} = \ln 9$$

$$-3x = \ln 9$$

$$x = \frac{\ln 9}{-3}$$

$$x \approx -.7324$$

$$3). 2e^{5x} - 8 = 30$$

$$2e^{5x} = 38$$

$$e^{5x} = 19$$

$$\ln e^{5x} = \ln 19$$

$$5x = \ln 19$$

$$x = \frac{\ln 19}{5}$$

$$x \approx .5889$$

Complete Got It? #3 p.479

a. ≈ 4.48 b. ≈ -2.3 c. $\approx .768$

Carbon dating is a method used to determine the age of organic material less than 50,000 years old. One of the formulas for carbon-14 dating is $t = \frac{\ln\left(\frac{N_f}{N_o}\right)}{-0.693} \times 5700$ where $\frac{N_f}{N_o}$ is the percent of carbon-14 remaining and t is the time in years. A fossil is found to have 30% carbon-14 compared to a living sample. Using this formula, what is the age of the fossil?

The fossil is approximately 9903 years old.

$$t = \frac{\ln\left(\frac{N_f}{N_o}\right)}{-0.693} \cdot 5700$$

$$t = \frac{\ln(.3)}{-0.693} \cdot 5700$$

$$t \approx 9902.81$$

Complete Got It? #4a p.480

No, the maximum velocity of 5.4 km/s is less than the 7.7 km/s needed for a stable orbit.

**Homework: p.481 #15, 16, 23, 24, 26-28, 32, 33, 35,
36, 38, 39, 41, 57**