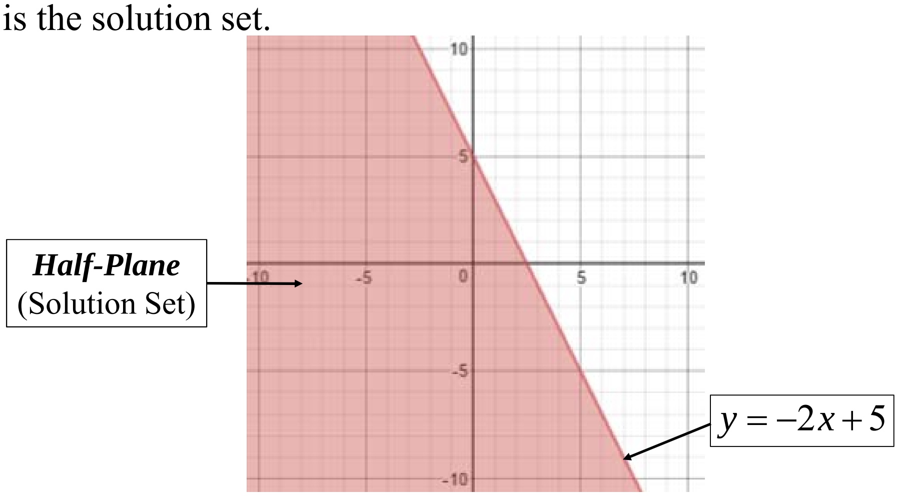
Notes 7.4 – Systems of Inequalities in Two Variables

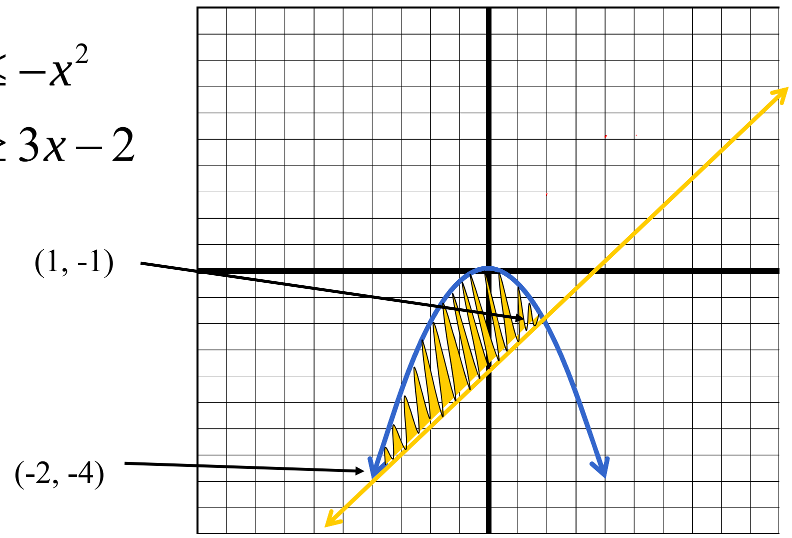
I. Half-Planes

Given the inequality $y \le -2x + 5$, the line $y = -2x + 5$ is the boundary, and the ***half-plane*** "below" the boundary is the solution set.



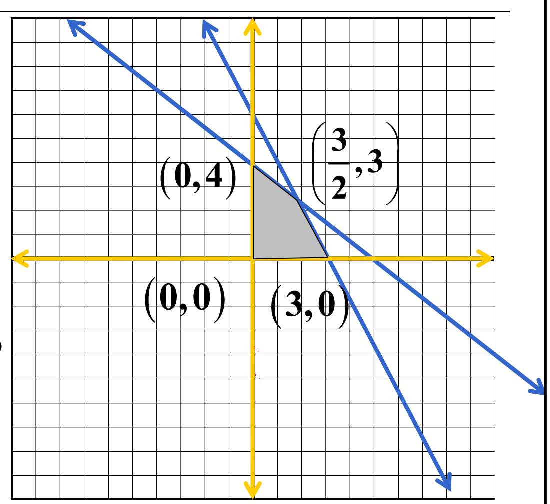
Ex.1 – Solve the following system

$$y \le -x^2$$
$$y \ge 3x - 2$$



Ex. 2– Solve the following system of inequalities.

$$2x + y \le 6$$
$$2x + 3y \le 12$$
$$x \ge 0$$
$$y \ge 0$$



The vertices for the solution to the system are:
$(0,0), (0,4), (3,0), \left(\frac{3}{2}, 2\right)$

The solution set is the area bounded by the lines through these points.

II. Linear Programming

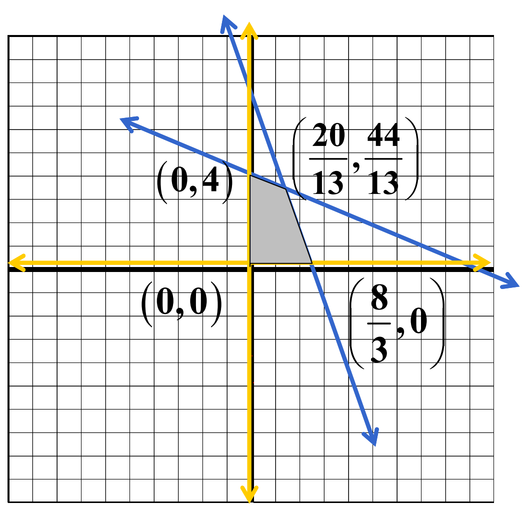
A.) Def. - Given an objective function f_n where

$$f = a_1x_1 + a_2x_2 + \ldots + a_nx_n.$$

In two dimensions, the function takes on the form $f = ax + by$. The solution is called the ***feasible region*** and the ***constraints*** are a system of inequalities.

Ex. 3–Find the maximum and minimum values of the objective function $f = 2x + 9y$ subject to the following constraints

$$y \le -\frac{2}{5}x + 4$$
$$y \le -3x + 8$$
$$x \ge 0$$
$$y \ge 0$$



$$f = 2x + 9y$$

(x, y)	$(0,0)$	$(0,4)$	$\left(\frac{8}{3}, 0\right)$	$\left(\frac{20}{13}, \frac{44}{13}\right)$
f	0	36	$\frac{16}{3}$	$\frac{436}{13}$

Minimum Value

Maximum Value

Ex. 4– Example 7 from the text on page 560

Johnson's Produce is purchasing fertilizer with two nutrients: N (nitrogen) and P (phosphorous). They need at least 180 units of N and 90 units of P. Their supplier has two brands of fertilizer for them to buy. Brand A costs \$10 a bag and has 4 units of N and 1 unit of P. Brand B costs \$5 a bag and has 1 unit of each nutrient. Johnson's Produce can pay at most \$800 for the fertilizer. How many bags of each brand should be purchased to minimize their cost?

Let x = # of bags of A and y = # of bags of B

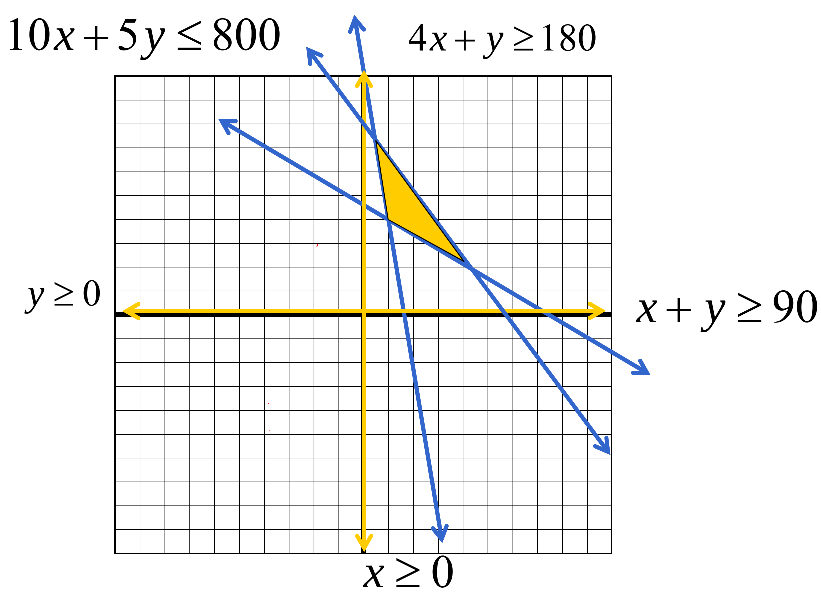
Therefore, our objective function for the COST is

$$C = 10x + 5y$$

with constraints of:

$$4x + y \geq 180$$
$$x + y \geq 90$$
$$10x + 5y \leq 800$$
$$x \geq 0, y \geq 0$$

Solution set looks like the following graph



The vertices of the region are

(x, y)	$(10, 140)$	$(70, 20)$	$(30, 60)$	
$C = 10x + 5y$	800	800	600	

Minimum Value