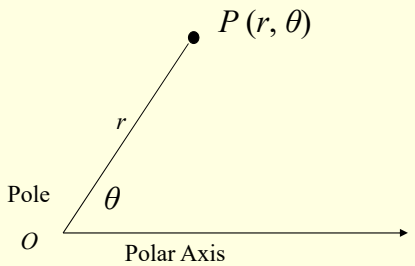
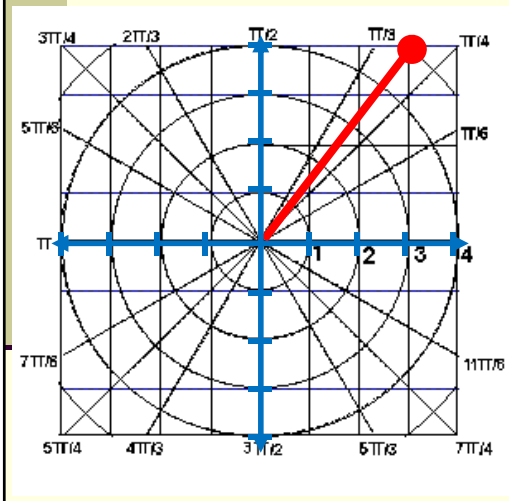
6.4 Polar Coordinates

I. Polar Coordinate System

- a plane with a point *O*, the *pole*, and a ray from *O,* the *polar axis.*
- Each point *P* in the plane is assigned *polar coordinates* (r, θ)as follows:
  - ***r**-directed distance* from O to P
  - ***θ**-directed angle* from the polar axis to $\overrightarrow{OP}$.



The Polar Grid looks as follows:



The point is graphed:

- Rectangular $P(3,4)$

(r,θ)

$$r=\sqrt{x^2+y^2}$$

$$=\sqrt{3^2+4^2}=5$$

$$\tan\theta=\frac{4}{3}$$

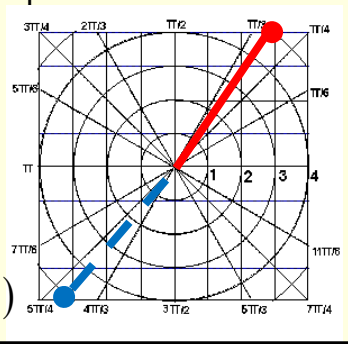
$$\theta=\tan^{-1}\frac{4}{3}\approx 53.1^\circ$$

- Polar $P(5,53.1^\circ)$

II. Finding All Polar Coordinates for a Point

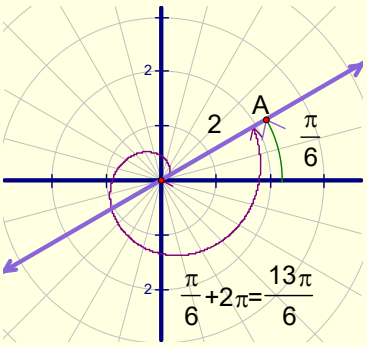
- *Rectangular coordinates* determine a unique point
- *Polar coordinates* of a point P in the plane are not unique.
  - $P(5,53.1^\circ)$ and $P(5,413.1^\circ)$ represent the same coordinate
    - ***Coterminal Angles***
  - $P(-5,53.1^\circ)$ and $P(5,233.1^\circ)$ represent the same coordinate

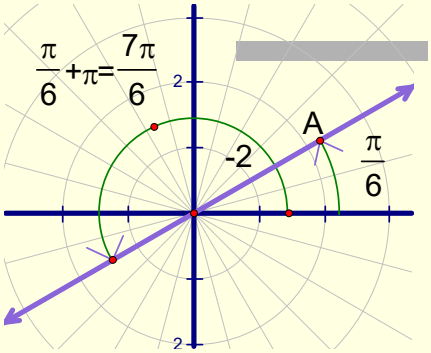
$P(-5,53.1^\circ)$
$P(5,53.1^\circ)$



$P(5,233.1^\circ)$

Find all polar coordinates that represent point A.





Generally, we have the following sets of points

$$\left(2,\frac{\pi}{6}+2n\pi\right)=\left(2,\frac{(12n+1)\pi}{6}\right) \qquad \left(-2,\frac{\pi}{6}+(2n+1)\pi\right)=\left(-2,\frac{(12n+7)\pi}{6}\right)$$

Given point P, find two other polar coordinates Q and R for P.

$$P\left(1,\frac{\pi}{3}\right) \qquad Q=\left(1,\frac{\pi}{3}+2\pi\right)=\left(1,\frac{7\pi}{3}\right)$$

$$R=\left(-1,\frac{\pi}{3}+\pi\right)=\left(-1,\frac{4\pi}{3}\right)$$

II. Finding All Polar Coordinates for a Point

- Let *P* have polar coordinates (r,θ). Any other polar coordinate of *P* must be of the form

$$\boxed{(r,\theta+2n\pi) \text{ or } (-r,\theta+(2n+1)\pi)}$$

Where *n* is any integer.

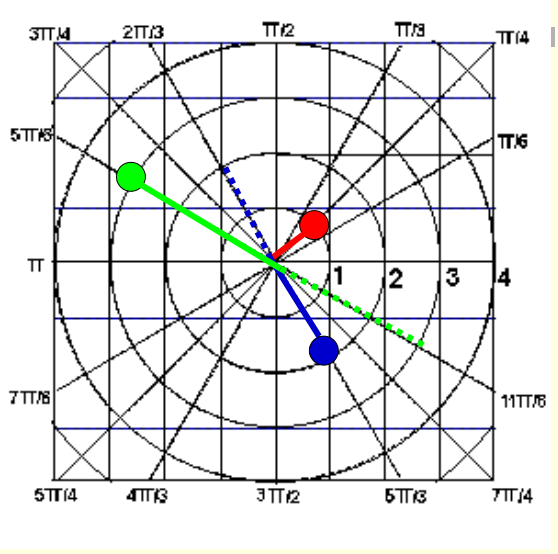
- In particular, the pole has polar coordinates $(0,\theta)$, where *theta* is any angle.

Plot the given polar coordinates.

1.) $P\left(1,\frac{\pi}{4}\right)$

2.) $Q\left(-2,\frac{2\pi}{3}\right)$

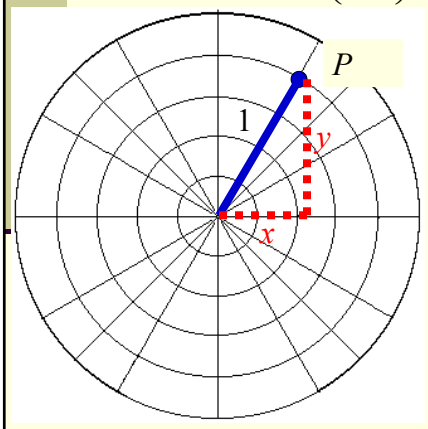
3.) $R\left(-3,-\frac{\pi}{6}\right)$



III. Coordinate Conversion

- Can we convert the polar coordinate to the Cartesian plane?

$P\left(1, \frac{\pi}{3}\right)$



Any ideas?

$\cos\left(\frac{\pi}{3}\right) = \frac{x}{1}$ $\quad \sin\left(\frac{\pi}{3}\right) = \frac{y}{1}$

$(1)\cos\left(\frac{\pi}{3}\right) = x$ $\quad (1)\sin\left(\frac{\pi}{3}\right) = y$

$x = \frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$

III. Coordinate Conversion

- Let P have the polar coordinates (r, θ) and rectangular coordinates (x, y), then:

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

Convert the following points from polar to rectangular form.

1.) $P\left(-3,\frac{5\pi}{4}\right)$

$x=-3\cos\frac{5\pi}{4}$

$y=-3\sin\frac{5\pi}{4}$

$\left(\frac{3}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$

2.) $Q\left(-2,\frac{5\pi}{6}\right)$

$x=-2\cos\frac{5\pi}{6}$

$y=-2\sin\frac{5\pi}{6}$

$\left(\sqrt{3},-1\right)$

Convert the following points from rectangular to polar form.

1.) $P(-1,1)$

$\tan\theta=\frac{1}{-1},\theta=\frac{3\pi}{4},\frac{7\pi}{4}$

$-1=r\cos\left(\frac{3\pi}{4}\right)$

$1=r\sin\left(\frac{3\pi}{4}\right)$

$\left(\sqrt{2},\frac{3\pi}{4}\right)$ or $\left(-\sqrt{2},\frac{7\pi}{4}\right)$

2.) $Q\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

$\tan\theta=-\sqrt{3}\quad\theta=\frac{2\pi}{3},\frac{5\pi}{3}$

$\frac{1}{2}=r\cos\left(\frac{5\pi}{3}\right)$

$-\frac{\sqrt{3}}{2}=r\sin\left(\frac{5\pi}{3}\right)$

$\left(1,\frac{5\pi}{3}\right)$ or $\left(-1,\frac{2\pi}{3}\right)$