6.4-Day 2
Polar Equations

IV. Equation Conversions

- Polar → rectangular

Given the following polar equation, how can we convert it to rectangular form?

$$r = 4\cos\theta$$

Remember our relations from yesterday?

$$x = r\cos\theta \qquad r^2 = x^2 + y^2$$

$$y = r\sin\theta$$

What if we multiply our equation by ***r***?

$$r^2 = 4r\cos\theta$$

Now, since $x = rcos\theta$ and $r^2 = x^2 + y^2$,
$r^2 = 4rcos\theta$ by substitution becomes: $x^2 + y^2 = 4x$

Finally, complete the square to finish the equation.

$$x^2 - 4x + y^2 = 0$$

$$x^2 - 4x + 4 + y^2 = 0 + 4$$

$$(x-2)^2 + y^2 = 4$$

Let's try another one… $r\cos\left(\theta + \frac{\pi}{3}\right) = 1$

$$r\cos\left(\theta + \frac{\pi}{3}\right) = r\left(\cos(\theta)\cos\left(\frac{\pi}{3}\right) - \sin(\theta)\sin\left(\frac{\pi}{3}\right)\right) = 1$$

$$r\cos\left(\theta + \frac{\pi}{3}\right) = \frac{1}{2}r\cos(\theta) - \frac{\sqrt{3}}{2}r\sin(\theta) = 1$$

$$\frac{1}{2}x - \frac{\sqrt{3}}{2}y = 1$$

$$x - \sqrt{3}y - 2 = 0$$

Rectangular → Polar

Given the following equation in rectangular form, how can we convert it to polar form?

$$(x-3)^2+(y-2)^2=13$$

$$x^2-6x+9+y^2-4y+4=13$$

$$x^2+y^2-6x-4y=0$$

$$r^2-6r\cos\theta-4r\sin\theta=0$$

$$r^2-6r\cos\theta-4r\sin\theta=0$$

$$r\left(r-6\cos\theta-4\sin\theta\right)=0$$

$r=0$

$r=6\cos\theta+4\sin\theta$ $\qquad$ $r=6\cos\theta+4\sin\theta$

Convert $y=2x-1$ to polar form.

$$y=2x-1$$

$$r\sin\theta=2\left(r\cos\theta\right)-1$$

$$r\sin\theta-2r\cos\theta=-1$$

$$r\left(\sin\theta-2\cos\theta\right)=-1$$

$$r=-\frac{1}{\sin\theta-2\cos\theta} \qquad r=\frac{1}{2\cos\theta-\sin\theta}$$