# 6.2: DOT PRODUCTS OF VECTORS

I. The Dot Product

*A. The dot product or inner product of the two vectors u and v below is:*

$$u = \langle u_1, u_2 \rangle, v = \langle v_1, v_2 \rangle$$

$$\boxed{u \cdot v = u_1 v_1 + u_2 v_2}$$

Note that this answer is a scalar

*B. Properties of the dot product*

Let *u*, *v*, and *w* be vectors and let *c* be a scalar.

$$\begin{array}{cc} u \cdot v = v \cdot u & u \cdot (v + w) = u \cdot v + u \cdot w \\ u \cdot u = |u|^2 & (u + w) \cdot v = u \cdot v + w \cdot v \\ 0 \cdot v = 0 & (cu) \cdot v = u \cdot (cv) = c(u \cdot v) \end{array}$$

C. *Example 1: find each dot product*

1. $\langle 3,2\rangle \cdot \langle -2,4\rangle \quad = 3(-2)+2(4)=\boxed{2}$

2. $\langle 1,\sqrt{2}\rangle \cdot \langle -3,-5\rangle$

$$=1(-3)+\sqrt{2}(-5)=\boxed{-3-5\sqrt{2}}$$

3. $(-i-2j)\cdot(4i-3j)$

$$=-1(4)+-2(-3)=\boxed{2}$$

D. *Example 2- Use the dot product to find the length of v.*

$$\boldsymbol{v}=\langle -8,15\rangle$$

$$|\boldsymbol{v}|=\sqrt{\boldsymbol{v}\cdot\boldsymbol{v}}=\sqrt{\langle -8,15\rangle\cdot\langle -8,15\rangle}$$

$$=\sqrt{64+225}$$

$$=\sqrt{289}$$

$$=17$$

## II. Angle Between Vectors

*A. Theorem: Angle Between Two Vectors*

If theta is the angle between the nonzero vectors *u* and *v*, then

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$$

*B. Example 3: Find the angle between the vectors u and v.*

$$u = \langle 1, -1 \rangle, v = \langle 3, 0 \rangle$$

$$\theta = \cos^{-1}\left(\frac{\langle 1, -1 \rangle \cdot \langle 3, 0 \rangle}{\sqrt{1^2 + (-1)^2}\sqrt{3^2 + 0^2}}\right)$$

$$\theta = \cos^{-1}\frac{3}{3\sqrt{2}} = \cos^{-1}\frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ = \frac{\pi}{4}$$

C. *Two vectors are Orthogonal (perpendicular) iff*

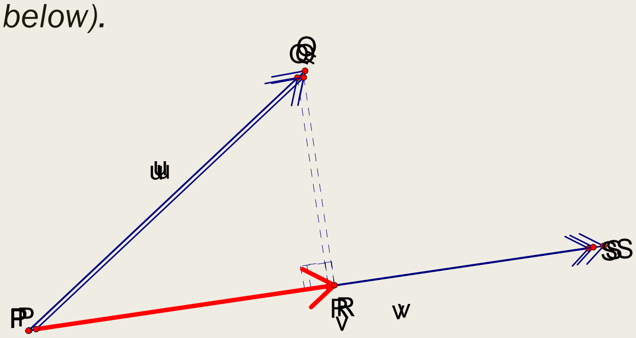
$$u \cdot v = 0$$

D. *Example 4: Prove that vectors u=3i+4j and v=4i-3j*

$$u \cdot v = 3(4) + 4(-3) = 0$$

III. Projecting One Vector onto Another

A. *The vector projection of vector u onto v is the vector determined by dropping a perpendicular from the terminal point of u to vector v (see below).*




$$\boxed{\overrightarrow{PR} = proj_v u}$$

(the vector projection of u onto v)

B. *Projection of u onto v.*

If *u* and *v* are nonzero vectors, the projection of *u* onto *v* is

$$proj_v u = \left(\frac{u \cdot v}{|v|^2}\right) v$$

C. *Example 5: Find the vector projection of vector u onto vector v. Then write u as the sum of two orthogonal vectors, one of which is the projection of u onto v.*

$$u = \langle -1, 3 \rangle, v = \langle 4, 4 \rangle$$

$$u_1 = proj_v u = \left(\frac{u \cdot v}{|v|^2}\right) v =$$

$$= \frac{8}{32}\langle 4, 4 \rangle = \boxed{\langle 1, 1 \rangle}$$

$$u_2 = u - u_1 \quad = \langle -1, 3 \rangle - \langle 1, 1 \rangle$$

$$= \boxed{\langle -2, 2 \rangle}$$