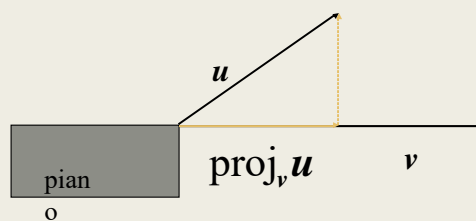


6.2 DOT PRODUCTS OF VECTORS

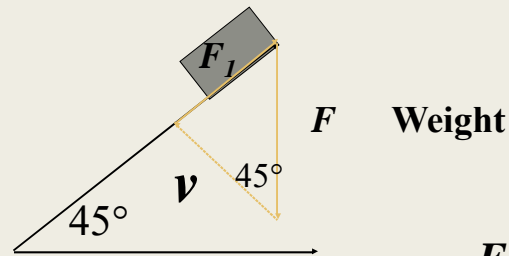
DAY 2

I. Pulling a Piano

A.) If we pull the piano with a force of \mathbf{u} , the effective force in the direction of \mathbf{v} is $\text{proj}_{\mathbf{v}}\mathbf{u}$



B.) Example 6 from the text- Find the effective force required to hold the boy and the sled in place on the hill if the combined weight of the boy and sled is 140 lbs.



$$F_1 = \text{proj}_{\mathbf{v}} F = \frac{\mathbf{F} \cdot \mathbf{v}}{|\mathbf{v}|^2} \mathbf{v}$$

$$F = \langle 0, -140 \rangle$$

$$\mathbf{v} = \langle \cos 45^\circ, \sin 45^\circ \rangle = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$F_1 = \text{proj}_{\mathbf{v}} F = \frac{\mathbf{F} \cdot \mathbf{v}}{|\mathbf{v}|^2} \mathbf{v}$$

$$\text{Magnitude} = 70\sqrt{2} \approx 99\text{lbs}$$

$$= -70\sqrt{2} \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \langle -70, -70 \rangle$$

II. Work

A.) If a force F is constant and in the same direction of \overrightarrow{AB} , then the work done by F moving from A to B is

$$w = (|F|)(|\overrightarrow{AB}|)$$

B.) Ex. – Find the work done by lifting a 200 lb. crate 2 feet off the ground.

$$w = |\langle 0, 200 \rangle| |\langle 0, 2 \rangle|$$

$$w = (200)(2) = 400 \text{ ft.-lbs.}$$

C.) Work in any direction –

$$w = \mathbf{F} \cdot \overrightarrow{AB} \quad w = (|F|)(|\overrightarrow{AB}|) \cos \theta$$

D.) Ex. – Find the work done by a 13 lb. force acting in the direction $\langle 2, 1 \rangle$ in moving an object from $(0, 0)$ to $(4, 0)$.

$$\mathbf{F} = 13 \frac{\langle 2, 1 \rangle}{|\langle 2, 1 \rangle|} = 13 \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle = \frac{13}{\sqrt{5}} \langle 2, 1 \rangle$$

$$\overrightarrow{AB} = \langle 4, 0 \rangle$$

$$w = \frac{13}{\sqrt{5}} \langle 2, 1 \rangle \cdot \langle 4, 0 \rangle = \frac{13}{\sqrt{5}} (8)$$

$$w = \frac{104}{\sqrt{5}} \approx 46.51 \text{ ft.-lbs.}$$