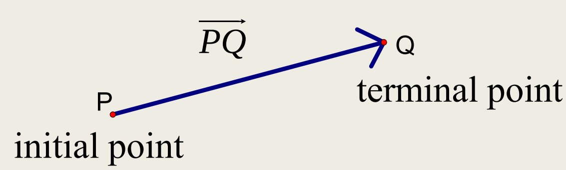
6-1: VECTORS IN THE PLANE

I. Directed Line Segments and Vectors

A. *Numbers that can be represented by a single real number that indicates magnitude or size are called* ***scalar quantities***.

B. *Other quantities that include a direction are called* ***directed line segments***.



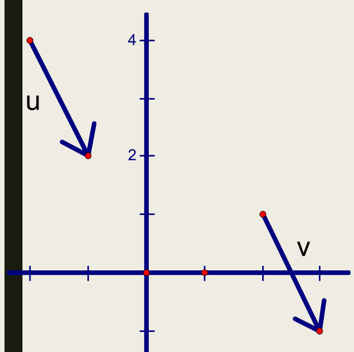
C. *The following represents the* ***length*** *or* ***magnitude*** *of* $\overrightarrow{PQ}$

$$|\overrightarrow{PQ}|$$

D. *Directed line segments with the same length and direction are* ***equivalent****.*

E. *We can represent directed line segments by one letter, and they are called vectors.*

F. *Example 1: Show that vectors* ***u*** *and* ***v*** *are equal.*



same direction

$$m_u = \frac{2-4}{-1-(-2)} = \boxed{-2} \qquad m_v = \frac{-1-1}{3-2} = \boxed{-2}$$

same magnitude

$$|u| = \sqrt{(2-4)^2 + [-1-(-2)]^2} = \boxed{\sqrt{5}}$$

$$|v| = \sqrt{(-1-1)^2 + (3-2)^2} = \boxed{\sqrt{5}}$$

II. Component Form of a Vector

A. *When a vector is placed with its initial point on the* ***origin****, it is in* ***standard position****.*

B. *If* ***v*** *is a vector in the plane equal to the vector with initial point (0,0)and terminal point (v_1, v_2), then the component form of* ***v*** *is*

$$v = \langle v_1, v_2 \rangle$$

C. *The numbers v_1 and v_2 are the components of* ***v****.*

D. *The vector $\langle v_1, v_2 \rangle$ is called the position vector of the point (v_1, v_2).*

D. *If* $v = \langle v_1, v_2 \rangle$ *is represented by the directed line segment* $\overrightarrow{PQ}$*, with initial point P(*x_1*,* y_1*) and terminal point Q(*x_2*,* y_2*), then the components of vector v are:*

$$v_1 = x_2 - x_1$$

$$v_2 = y_2 - y_1$$

$$\boxed{v = \langle x_2 - x_1, y_2 - y_1 \rangle}$$

E. *Definition: Magnitude or Length*

The *magnitude* or *length* of the vector $v = \overrightarrow{PQ}$ determined by *P(*x_1*,* y_1*)* and *Q(*x_2*,* y_2*)* is

$$|v| = \sqrt{v_1^2 + v_2^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

F. *Two vectors* $\langle a,b \rangle$ *and* $\langle c,d \rangle$ *are equal iff a=c and b=d.*

G. *The vector* $\langle 0,0 \rangle$ *with length 0 and no direction is the zero vector and is denoted 0.*

H. *Example 2: Find the component form and magnitude of the vector with endpoints P(-1,5) and Q(3,1).*

$$v = \langle 3-(-1), 1-5 \rangle = \boxed{\langle 4, -4 \rangle}$$

$$|v| = \sqrt{4^2 + (-4)^2} = \sqrt{32} = \boxed{4\sqrt{2}}$$

III. Vector Operations

A. Vector Addition and Scalar Multiplication:

Let $u=\langle u_1,u_2\rangle$ and $v=\langle v_1,v_2\rangle$ be vectors and *k* be a real number (scalar).

1. The *sum of vectors u* and *v* is the vector

$$u+v=\langle u_1,u_2\rangle+\langle v_1,v_2\rangle=\langle u_1+v_1,u_2+v_2\rangle$$

2. The *product of the scalar k and the vector u* is

$$ku=k\langle u_1,u_2\rangle=\langle ku_1,ku_2\rangle$$

B. Example 3: Perform the following vector operations for vectors u and v below

$$u=\langle -2,1\rangle v=\langle 3,4\rangle$$

1. $u+v=\langle -2+3,1+4\rangle=\boxed{\langle 1,5\rangle}$

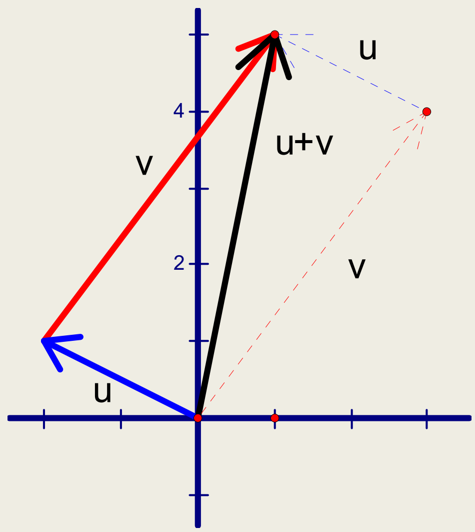
2. $2u=2\langle -2,1\rangle=\boxed{\langle -4,2\rangle}$

3. $(-1)u+2v=\langle 2,-1\rangle+\langle 6,8\rangle$

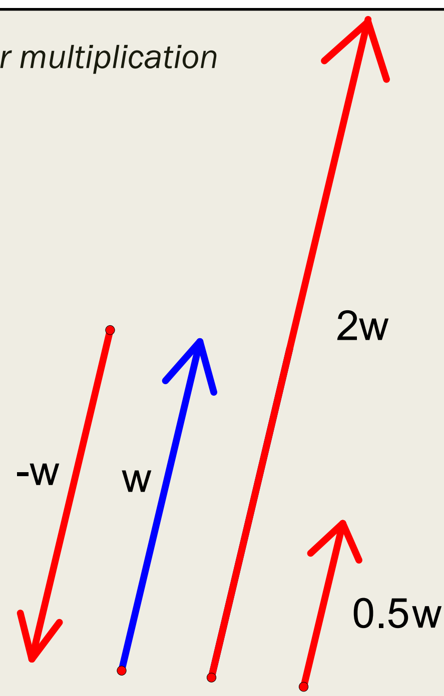
$$=\boxed{\langle 8,7\rangle}$$

C. Graphical Vector Addition

$$u+v=\langle -2,1\rangle+\langle 3,4\rangle$$



D. Vector multiplication



IV. Unit Vectors

A. *A vector with length $|u|=1$ is a unit vector. If v is not the zero vector $\langle 0,0\rangle$, then the vector*

$$\boxed{u=\frac{v}{|v|}=\frac{1}{|v|}v}$$

Is a ***unit vector*** in the direction of *v*.

B. *Example 4: Find a unit vector in the direction of vector $v=\langle 4,-1\rangle$, and verify that it has a length of 1.*

$$|v|=|\langle 4,-1\rangle|=\sqrt{4^2+(-1)^2}=\sqrt{17}$$

$$\boxed{\frac{v}{|v|}=\left\langle \frac{4}{\sqrt{17}},-\frac{1}{\sqrt{17}}\right\rangle}$$

$$\left|\left\langle \frac{4}{\sqrt{17}},-\frac{1}{\sqrt{17}}\right\rangle\right|=\sqrt{\left(\frac{4}{\sqrt{17}}\right)^2+\left(-\frac{1}{\sqrt{17}}\right)^2}$$

$$=\sqrt{\frac{16}{17}+\frac{1}{17}}$$

$$=\sqrt{\frac{17}{17}}=1$$

C. *The two unit vectors below are the standard unit vectors.*

$$i = \langle 1,0 \rangle ;\ j = \langle 0,1 \rangle$$

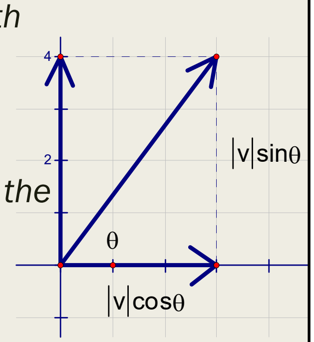
D. *Any vector can be written in terms of the standard unit vectors.*

$$v = \langle a,b \rangle = ai + bj$$

E. *The values a and b are the horizontal and vertical components of vector v.*

V. Direction Angles

A. *The angle that a vector makes with the positive x-axis is its direction angle.*



B. *Note that we can write vector v in the following form:*

$$v = \left(|v|\cos\theta\right)i + \left(|v|\sin\theta\right)j$$

C. *The unit vector in the direction of v is:*

$$u = \frac{v}{|v|} = \left(\cos\theta\right)i + \left(\sin\theta\right)j$$

D. *Example 5: Find the magnitude and direction of the following vector.*

$$w = \langle 5, -3 \rangle$$

$$|w| = \sqrt{5^2 + (-3)^2} = \sqrt{34}$$

$$\theta = \tan^{-1}\left(\frac{-3}{5}\right) \approx -30.96$$

note that this angle is in the 4th quadrant, which is where the point (5,-3) is, so we don't need to change the angle.

E. *Example 6: write the vector in component form with direction angle -160 degrees and magnitude 12.*

$$\boxed{v = 12\cos(-160^\circ)\,i + 12\sin(-160^\circ)\,j}$$

$$\boxed{v \approx -11.276i - 4.104j}$$