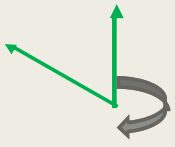
6.1 VECTORS IN THE PLANE – DAY 2 APPLICATIONS

I. Velocity Vectors

A.) **Velocity** - A vector quantity because it has both magnitude and direction. Speed is the magnitude.

B.) **Example 1** – A plane leaves Newark International Airport on a bearing of 295 degrees. If the plane is traveling at a speed of 500 mph, write the plane's velocity in component form.



$$B = 295°$$

$$\therefore \theta = 360 - 295 + 90 = 155°$$

$$\mathbf{v} = 500\langle \cos 155, \sin 155 \rangle$$

$$\mathbf{v} \approx \langle -453.154, 211.309 \rangle$$

C.) **Example 2** – The plane in Example 1 is being aided by a 50-mph wind with a bearing of 310 degrees. Calculate the resultant compass heading and ground speed of the plane.

$B_w = 310°$

$\therefore \theta = 360 - 310 + 90 = 140°$

$\mathbf{v}_w = 50\langle \cos 140, \sin 10 \rangle$

$\mathbf{v}_w \approx \langle -38.302, 32.139 \rangle$

$\mathbf{v}_p + \mathbf{v}_w =$

$\langle -453.154, 211.309 \rangle + \langle -38.302, 32.139 \rangle =$

$\langle -491.456, 243.448 \rangle$

$\text{Speed} = |\langle -491.456, 243.448 \rangle|$

$= \sqrt{(-491.456)^2 + (243.448)^2}$

$\approx 548.449 \text{ mph}$

$\theta = \cos^{-1}\left(\frac{-491.456}{548.449}\right)$

$\approx 153.684°$

$B \approx 360 - (153.684 - 90)$

$\approx 296.352°$

D.) **Example 3** – At what bearing and speed should a pilot head if he wants to fly at 450 mph on a bearing of 300 degrees, but there is a 40 mph coming from the west, heading due east?

$\mathbf{v}_p + \mathbf{v}_w = 450\langle \cos(150°), \sin(150°) \rangle$

$\mathbf{v}_p = k\langle \cos\theta, \sin\theta \rangle$

$\mathbf{v}_w = \langle 40, 0 \rangle$

$\langle k\cos\theta, k\sin\theta \rangle + \langle 40, 0 \rangle = \langle -389.711, 225 \rangle$

$k\cos\theta + 40 = -389.711$

$k\sin\theta + 0 = 225$

$k\sin\theta = 225$

$k\cos\theta = -429.711$

$\tan\theta = \frac{225}{-429.711}$

$\theta = -27.637 + 180 \approx 152.363°$

$B_p = 360 - (152.363 - 90)$

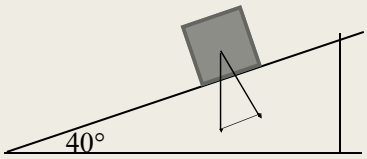
$B_p \approx 297.637°$

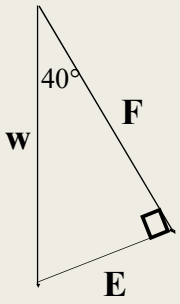
$k\sin(152.363°) = 225$

$k \approx 485.052 \text{ mph}$

II. Gravity

A.) **Example 4** – A box on an inclined plane weighs 75 lb, and the plane makes an angle of 40 degrees with the horizontal. What is the force exerted on the plane by the box?





$\mathbf{w}$ = The weight vector

$\mathbf{F}$ = Force $\perp$ to plane

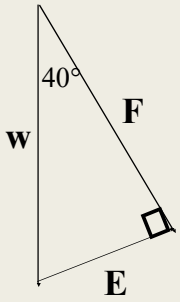
$\mathbf{E}$ = Effective Force to hold in place

θ = Angle if incline of the ramp, opposite $\mathbf{E}$

$\theta = 40°$

$|\mathbf{w}| = 75$

$|\mathbf{F}| = 75\cos(40) \approx \boxed{57.453 \text{ lb}}$



B.) **Example 5** – What is the effective force needed to keep the box from sliding down the plane?

$\theta = 40°$

$|\mathbf{w}| = 75$

$|\mathbf{E}| = 75\sin(40) \approx \boxed{48.209 \text{ lb}}$