5-7: The Binomial Theorem

Algebra 2

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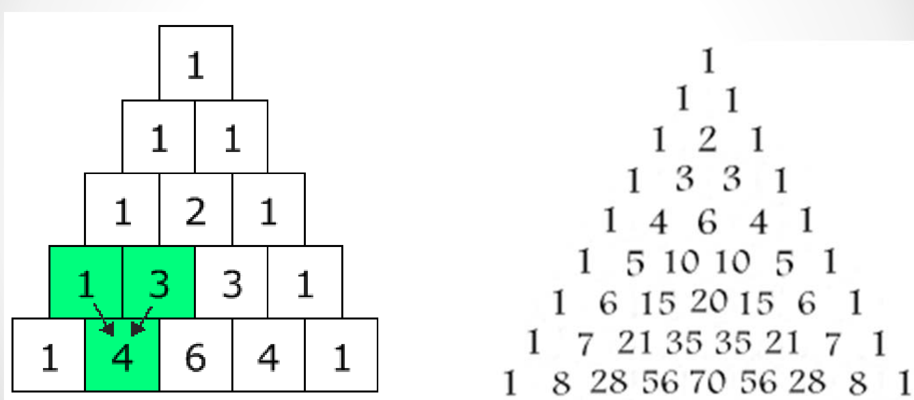
Pascal's Triangle

Named after French mathematician Blaise Pascal.



Each row begins and ends with 1 and the other numbers are the sum of the numbers above it.

Expand the following binomials:

$(a+b)^0$ — 1

$(a+b)^1$ — $1a^1+1b^1$

$(a+b)^2$ — $1a^2+2a^1b^1+1b^2$

$(a+b)^3$ — $1a^3+3a^2b^1+3a^1b^2+1b^3$

$(a+b)^4$ — $1a^4+4a^3b^1+6a^2b^2+4a^1b^3+1b^4$

Look at the coefficients only and find what the pattern is.

Pascal's Triangle with the binomial exponent corresponding to row number in Pascal's Triangle

The Binomial Theorem

For every positive integer n:

$(a+b)^n = P_0a^n + P_1a^{n-1}b^1 + P_2a^{n-2}b^2 + \cdots + P_{n-1}ab^{n-1} + P_nb^n$
where $P_0, P_1, \ldots, P_n$ are the numbers in the nth row of Pascal's Triangle.

Use Pascal's Triangle to expand $(a+b)^5$

The 5th row of Pascal's triangle is 1 5 10 10 5 1.
These become the coefficients of the expansion:

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Use Pascal's Triangle to expand $(x + 3)^4$:

From the 4th row of Pascal's Triangle: 1 4 6 4 1

The binomial takes the form $(a + b)^4$, so $a = x$ and $b = 3$

Write the binomial expansion using $(a + b)^4$, then substitute the values for a and b:

$$(a+b)^4 = 1a^4 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1b^4$$

$$(x+3)^4 = 1x^4 + 4x^3\left(3^1\right) + 6x^2\left(3^2\right) + 4x^1\left(3^3\right) + 1\left(3^4\right)$$

$$= x^4 + 12x^3 + 54x^2 + 108x^2 + 81$$

Use Pascal's Triangle to expand $(3 - z)^3$:

From the 3rd row of Pascal's Triangle: 1 3 3 1

The binomial takes the form $(a + b)^3$, so $a = 3$ and $b = -z$

Write the binomial expansion using $(a + b)^3$, then substitute the values for a and b:

$$(a+b)^3 = 1a^3 + 3a^2b^1 + 3a^1b^2 + 1b^3$$

$$(3-z)^3 = 1(3)^3 + 3(3)^2(-z)^1 + 3(3)^1(-z)^2 + 1(-z)^3$$

$$= 27 - 27z + 9z^2 - z^3$$

Homework: p.329 #9-23 odd, 54-56, 59-65 odd