5-6: The Fundamental Theorem of Algebra

Algebra 2
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The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly *n* roots, including multiple and complex roots.

- A polynomial with no constant term has 0 as one of its roots.

  $f(x) = 3x^3 + x^2 - 5x$ Has GCF of *x*, therefore $x = 0$

- A polynomial with *odd* degree and real coefficients must have a root in the set of real numbers (rational or irrational)

  Graph an example

- A polynomial with an *even* degree doesn't have to cross the *x*-axis.

  Graph an example

- A polynomial with an *odd* degree must cross the *x*-axis.

  Graph an example

What are the zeroes of $f(x) = x^4 + 2x^3 - 4x^2 - 7x - 2$?

Graph the function and find any real roots: $x = -1, 2$

Use synthetic division until you have a quadratic function:

$$\begin{array}{r|rrrrr} 2 & 1 & 2 & -4 & -7 & -2 \\ & & 2 & 8 & 8 & 2 \\ \hline & 1 & 4 & 4 & 1 & 0 \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & 4 & 4 & 1 \\ & & -1 & -3 & -1 \\ \hline & 1 & 3 & 1 & 0 \end{array}$$

Use the quadratic formula to find the rest of the roots:

$$x = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$$

The four roots are: $x = -1, 2, \dfrac{-3 \pm \sqrt{5}}{2}$

Homework: p.322 #8, 9, 12, 13, 16, 17, 22, 24, 25, 50-52, 54, 58, 59