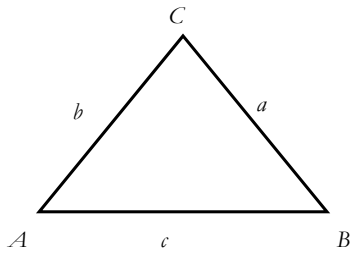
5.5 Law of Sines

I. Law of Sines

In any triangle with opposite sides a, b, and c:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Sines is used to solve any triangle where you are given, in order, AAS, ASA, and SSA. Any triangle where we are given SAS must be solved by using the Law of Cosines.

II. Examples

Solve the following triangles:

A. $\angle A = 49°, \angle B = 46°, a = 5$

$\angle C = 180 - (46 + 49) = 85°$

$$\frac{\sin 49}{5} = \frac{\sin 46}{b}$$

$$b \approx 4.766$$

$$\frac{\sin 49}{5} = \frac{\sin 85}{c}$$

$$c \approx 6.600$$

B. $\angle A = 49°, \angle B = 46°, c = 5$

$\angle C = 180 - (46 + 49) = 85°$

$$\frac{\sin 49}{a} = \frac{\sin 85}{5}$$

$$a \approx 3.788$$

$$\frac{\sin 46}{b} = \frac{\sin 85}{5}$$

$$b \approx 3.610$$

C.) $\angle A = 30°,\ a = 6,\ b = 7$

$$\frac{\sin 30}{6} = \frac{\sin B}{7} \qquad \angle B \approx \sin^{-1}\left(\frac{7 \sin 30}{6}\right) \approx 35.685°$$

$\angle C = 180 - (30 + 35.685) = 114.315°$

$$\frac{\sin 114.315}{c} = \frac{\sin 30}{6}$$

$$c = \frac{6 \sin(114.315)}{\sin 30} \approx 10.936$$

But… Couldn't B be obtuse? The inverse sin only gives values in the first and fourth quadrant, therefore, if the angle is given by inverse sin is less than 90 degrees and there is an SSA situation, angle B could be obtuse. Therefore,

$$\angle B = 180 - (35.685) = 144.315°$$

$$\angle C = 180 - (144.315 + 30) = 5.685°$$

$$\frac{\sin 5.685}{c} = \frac{\sin 30}{6}$$

$$c = \frac{6\sin(5.685)}{\sin 30} \approx 1.189$$

III. The Ambiguous Case - SSA

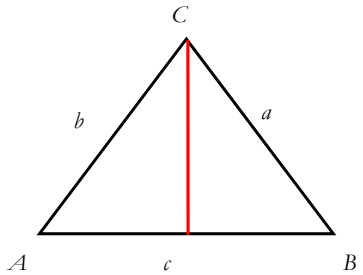
Two possibilities given Angle A and side a:

A.) If a is greater than b, then there is only **ONE** solution.

B.) If a is less than b,

1.) there are **TWO** possible triangles if $h < a$.

2.) **NO** possible triangles if $h > a$.



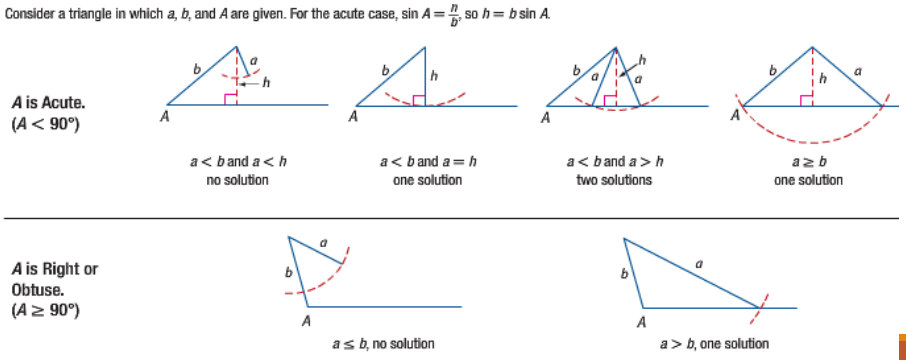
SSA Triangles:

Does not define a unique triangle

- Called the **ambiguous case**

Three possible answers:

- No solution, one solution or two solutions



II. Applications

Two lighthouses at points *X* and *Y* are 40 kilometers apart. Each has a visual contact with a fishing boat at point *Z*. If the angles are as given below, how far is the fishing boat from lighthouse *Y*?

$$m\angle ZXY = 20^\circ 30', m\angle ZYX = 115^\circ$$

$$m\angle Z = 180 - 20^\circ 30' - 115^\circ = 44^\circ 30'$$

$$\frac{\sin 44^\circ 30'}{40} = \frac{\sin 115}{y}$$

$$y = \frac{40 \sin 115}{\sin 44^\circ 30'} \approx \boxed{51.72 km}$$