5.4 Multiple Angle Identities

I. Double-Angle Identities

A.) $\sin(2u) = 2\sin u \cos u$

B.) $\cos(2u) = \cos^2 u - \sin^2 u$

$= 2\cos^2 u - 1$

$= 1 - 2\sin^2 u$

C.) $\tan(2u) = \dfrac{2\tan u}{1 - \tan^2 u}$

II. Power-Reducing Identities

$$\text{A.)}\ \sin^2 u = \frac{1-\cos(2u)}{2}$$

$$\text{B.)}\ \cos^2 u = \frac{1+\cos(2u)}{2}$$

$$\text{C.)}\ \tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)}$$

III. Half-Angle Identities

$$\text{A.)}\ \sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\text{B.)}\ \cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\text{C.)}\ \tan\left(\frac{u}{2}\right) = \begin{cases} \pm\sqrt{\dfrac{1-\cos u}{1+\cos u}} \\ \dfrac{1-\cos u}{\sin u} \\ \dfrac{\sin u}{1+\cos u} \end{cases}$$

IV. Examples

Derive all the multiple angle identities.

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos(u+u) =$$

$$\cos u \cos u - \sin u \sin u =$$

$$\cos^2 u - \sin^2 u = \cos^2 u - \sin^2 u$$

Solve the following equation on [0, 2π). Give exact answers.

$$\sin 2x = \cos 3x$$

$$\sin 2x - \cos 3x = 0$$

$$2\sin x \cos x - \left(\cos\left(2x + x\right)\right) = 0$$

$$2\sin x \cos x - \left(\cos 2x \cos x - \sin 2x \sin x\right) = 0$$

$$2\sin x \cos x - \cos 2x \cos x + \sin 2x \sin x = 0$$

$$2\sin x \cos x - \left(1 - 2\sin^2 x\right)\cos x + 2\sin^2 x \cos x = 0$$

$$\cos x\left(2\sin x - 1 + 2\sin^2 x + 2\sin^2 x\right) = 0$$

$$\cos x\left(4\sin^2 x + 2\sin x - 1\right) = 0$$

$$\cos x = 0 \quad \text{or} \quad \left(4\sin^2 x + 2\sin x - 1\right) = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$\sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$x = \sin^{-1}\left(\frac{-1 \pm \sqrt{5}}{4}\right)$$