5-4: DIVIDING POLYNOMIALS

Algebra 2
Mr. Gallo

POLYNOMIAL LONG DIVISION

- Numerical

$$22\overline{)770}$$

- Polynomial

$$2x+1\overline{)6x^2+7x+2}$$

The remainder for each problem is 0, so 22 is a factor of 770 and 2x+1 is a factor of $6x^2+7x+2$.

USING POLYNOMIAL LONG DIVISION

Use long division to divide $5x^2+2x+3$ by $x+1$.

$$\begin{array}{r} 5x-3 \\ x+1\overline{)\,5x^2+2x+3} \\ -\;\;5x^2+5x \\ \hline -3x+3 \\ -\;\;-3x-3 \\ \hline 6 \end{array}$$

The *degree* is less than x +1 so this is the *Remainder*

Divide: $\frac{5x^2}{x}=5x$

Multiply: $5x(x+1)=5x^2+5x$

Subtract to get $-3x$. Bring down 3.

Divide: $\frac{-3x}{x}=-3$

Multiply: $-3(x+1)=-3x-3$

Subtract to get 6.

The *quotient* is $5x-3, R6$.

Complete Got It? #1 p. 304

$3x-8,\ \mathrm{R}0$

DIVISION ALGORITHM FOR POLYNOMIALS

You can divide polynomial *P(x)* by polynomial *D(x)* to get polynomial quotient *Q(x)* and polynomial remainder *R(x)*. The result is *P(x)=D(x)Q(x)+R(x)*.

$$\begin{array}{r} Q(x) \\ D(x)\overline{)\,P(x)} \\ \vdots \\ \hline R(x) \end{array}$$

If *R(x)=0*, then *P(x)=D(x)Q(x)* and *D(x)* and *Q(x)* are factors of *P(x)*.

To use long division, *P(x)* and *D(x)* should be in standard form with zero coefficients where appropriate. The process stops when the degree of the remainder, *R(x)*, is less than the degree of the divisor, *D(x)*.

CHECKING FACTORS

Is x^2+1 a factor of $3x^4-4x^3+12x^2+5$?

$$
\begin{array}{r}
3x^2 - 4x + 9 \\
x^2+0x+1 \overline{\big) 3x^4-4x^3+12x^2+0x+5} \\
-\underline{3x^4+0x^3+3x^2} \\
-4x^3+9x^2+0x \\
-\underline{-4x^3+0x^2-4x} \\
9x^2-4x+5 \\
-\underline{9x^2+0x+9} \\
-4x-4
\end{array}
$$

The *degree* is less than the divisor so this is the *Remainder*

The quotient is: $3x^2-4x+9,\ R-4x-4$

The remainder is not 0 so x^2+1 is not a factor of $3x^4-4x^3+12x^2+5$

Is x^2-2 a factor of $P(x)=x^4-x^2-2$? If it is, write *P(x)* as a product of two factors.

$$
\begin{array}{r}
x^2+0x+1 \\
x^2+0x-2 \overline{\big) x^4+0x^3-x^2+0x-2} \\
-\underline{x^4+0x^3-2x^2} \\
x^2+0x-2 \\
-\underline{x^2+0x-2} \\
0
\end{array}
$$

$P(x)=(x^2+1)(x^2-2)$

HOMEWORK: p.308 #9-16, 72-78 even

p.308 #9-16, 72-78 even

9. quotient: $x - 8$

10. quotient: $3x - 5$

11. quotient: $x^2 + 4 + 3$, R 5

12. quotient: $2x^2 + 5x + 2$

13. quotient: $3x^2 + 3x + 2$

14. quotient: $9x - 12$, R -32

15. quotient: $x - 10$, R 40

16. quotient: $x^2 + 4x + 3$

72. The real solutions are 0 and 1.

74. $\frac{-3 \pm \sqrt{17}}{2}$

76. $1, \ -\frac{5}{7}$

78. $3 \pm \sqrt{2}$

SYNTHETIC DIVISION

- Can only use synthetic division when you divide a polynomial by "x – a"
- Write out ALL coefficients (including zeroes!) of polynomial when in standard form.
- For a divisor, use "a." (Reverse the sign of the number in "x – a.")

EXAMPLE 1

$$(x^3 - 14x^2 + 51x - 54) \div (x + 2)$$

-2	1	-14	51	-54
		-2	32	-166
	1	-16	83	-220

Final Number in Bottom Row is the Remainder

The quotient is $x^2 - 16x + 83, R - 220$

EXAMPLE 2

$$(x^3 - 57x + 56) \div (x - 7)$$

7	1	0	-57	56
		7	49	-56
	1	7	-8	0

The quotient is $x^2 + 7x - 8, R0$

REMAINDER THEOREM

- If you divide a polynomial P(x) by x – a, then the remainder is P(a).

Reason:

$$P(x) = D(x)Q(x) + R(x)$$

$$P(x) = (x-a)Q(x) + R(x)$$

$$P(a) = (a-a)Q(a) + R(a)$$

$$P(a) = 0 + R(a)$$

$$P(a) = R(a)$$

EXAMPLE 3

- Find P(3) for $P(x) = x^5 - 2x^3 - x^2 + 2$ using synthetic division.

3	1	0	-2	-1	0	2
		3	9	21	60	180
	1	3	7	20	60	182

$$P(3) = 182$$

Complete Got It? #5 p. 307 $P(-4) = 0$

HOMEWORK: p.308 #21-27odd, 33-39 odd, 71-79 odd