5-3: SOLVING POLYNOMIAL EQUATIONS

Algebra 2
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FINDING REAL ROOTS BY GRAPHING

Find the real solutions to $2x^3 + 5 = 3x^2 - 2x$.

- Method 1:
  - Graph each equation in **Y=**
    - $Y_1 = 2x^3 + 5$
    - $Y_2 = 3x^2 - 2x$
  - Use the **INTERSECT** feature to find points of intersection

- Method 2:
  - Rewrite the equation as $2x^3 - 3x^2 + 2x + 5 = 0$.
  - Graph the related function and use the **ZERO** feature.

What are the real solutions to $x^3 + x^2 = x - 1$?

- Method 1
  - $Y_1 = x^3 + x^2$
  - $Y_2 = x - 1$
  - Use INTERSECTION
  - The solution is: -1.84
- Method 2
  - $Y_1 = x^3 + x^2 - x + 1$
  - Use ZERO
  - The solution is: -1.84

Which method seems easier and more reliable? Why?

SPECIAL CASE FACTORING

- Sum of Cubes:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$x^3 + 27 = (x+3)(x^2 - 3x + 9)$

$a = \sqrt[3]{x^3} = x$

$b = \sqrt[3]{27} = 3$

- Difference of Cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$x^3 - 27 = (x-3)(x^2 + 3x + 9)$

$a = \sqrt[3]{x^3} = x$

$b = \sqrt[3]{27} = 3$

Complete the following: $x^6 + 64 = (x^2 + 4)(x^4 - 4x^2 + 16)$

$8x^6 - 125 = (2x^2 - 5)(4x^4 + 10x^2 + 25)$

SOLVING POLYNOMIALS USING FACTORS

- If *(x-a)* is a factor of a polynomial, then the polynomial =0 when *x=a*.
- To solve using factors:
  1. Write the equation in the form *P(x)=0*.
  2. Factor *P(x)*.
  3. Use the Zero Product Property to solve.

$$4x^3 - 6x^2 = 4x$$
$$4x^3 - 6x^2 - 4x = 0$$
$$2x(2x^2 - 3x - 2) = 0$$
$$2x(2x+1)(x-2) = 0$$

$2x = 0$ $(2x+1) = 0$ $(x-2) = 0$

$x = 0$ $2x = -1$ $x = 2$

$x = -\frac{1}{2}$

The solutions are: $-\frac{1}{2}, 0, 2$

What are the solutions of each polynomial equation over the complex numbers?

$$2x^3 = -54$$
$$2x^3 + 54 = 0$$
$$2\left(x^3 + 27\right) = 0$$
$$x^3 + 27 = 0$$
$$\left(x+3\right)\left(x^2 - 3x + 9\right) = 0$$

$$x^3 = 8x - 2x^2$$
$$x^3 + 2x^2 - 8x = 0$$
$$x\left(x^2 + 2x - 8\right) = 0$$
$$x\left(x+4\right)\left(x-2\right) = 0$$

The solutions are:

$x = -3$ $x = \frac{3 \pm 3i\sqrt{3}}{2}$

The solutions are:

$x = 0$ $x + 4 = 0$ $x - 2 = 0$

$x = -4$ $x = 2$

What are the solutions of each polynomial equation over the complex numbers?

$$x^4 - 5x^2 = 16$$

$$x^4 - 5x^2 - 16 = 0$$

$a = x^2$ $\quad a^2 - 5a - 16 = 0$

$$(a-8)(a+2) = 0$$

$$(x^2-8)(x^2+2) = 0$$

The solutions are:
$x = \pm 2\sqrt{2} \qquad x = \pm i\sqrt{2}$

$$2x^4 - 20x^2 + 50 = 0$$

$$2(x^4 - 10x^2 + 25) = 0$$

$$x^4 - 10x^2 + 25 = 0$$

$a = x^2$ $\quad a^2 - 10a + 25 = 0$

$$(a-5)^2 = 0$$

$$(x^2-5)^2 = 0$$

The solutions are: $x = 0$

What are three consecutive even integers whose product is 4 times their sum?

$x = 1^{st}$ integer $\qquad x+2 = 2^{nd}$ integer $\qquad x+4 = 3^{rd}$ integer

$$4(x + x + 2 + x + 4) = x(x+2)(x+4)$$

$$4x + 4x + 8 + 4x + 16 = x(x^2 + 6x + 8)$$

$$12x + 24 = x^3 + 6x^2 + 8x$$

$$0 = x^3 + 6x^2 + 8x - 12x - 24$$

$$0 = x^3 + 6x^2 - 4x - 24$$

The zeros are: $-6, -2, 2$

The answers are: 2, 4, 6 and -2, -4, -6

Homework: p. 301 #11-37 odd