5.2 Proving Trig Identities

I. General Strategies

1.) Begin with the more complicated expression on one side of the identity.

2.) Work on ONLY ONE SIDE of the identity!!!

3.) Each step should consist of an expression easily seen from the step before.

4.) Always be mindful of favorable manipulations.

A.) Common denominators.

B.) Use $(a + b)(a - b) = (a^2 - b^2)$ to set up Pyth. Ident.

C.) Convert to sin and cos (if nothing else!)

5.) End with the expression on the other side of the identity.

Prove: $\sec x \csc x = \tan x + \cot x$

$$\sec x \csc x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$\sec x \csc x = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\sec x \csc x = \frac{1}{\sin x \cos x}$$

$$\sec x \csc x = \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$$

$$\sec x \csc x = \sec x \csc x$$ ✓✓

Prove: $\dfrac{1}{\sec x - 1} + \dfrac{1}{\sec x + 1} = 2\cot x \csc x$

$$\frac{\sec x + 1}{(\sec x - 1)(\sec x + 1)} + \frac{\sec x - 1}{(\sec x - 1)(\sec x + 1)} =$$

$$\frac{2\sec x}{\sec^2 x - 1} =$$

$$\frac{2\sec x}{\tan^2 x} =$$

$$\frac{\frac{2}{\cos x}}{\frac{\sin^2 x}{\cos^2 x}} = 2\cot x \csc x$$

$$\frac{2\cos x}{\sin^2 x} =$$

$$\frac{\left(\frac{2}{\cos x}\right)\left(\frac{\cos^2 x}{\sin^2 x}\right)}{\left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{\cos^2 x}{\sin^2 x}\right)} =$$

$$\left(\frac{2\cos x}{\sin x}\right)\left(\frac{1}{\sin x}\right) =$$

$$2\cot x \csc x = 2\cot x \csc x$$

Prove: $\frac{\cot^2 u}{1+\csc u} = \cot u\left(\sec u - \tan u\right)$

$$\cot u\left(\frac{\cot u}{1+\csc u}\right) =$$

$$\cot u\left(\frac{\cot u\left(1-\csc u\right)}{-\cot^2 u}\right) =$$

$$\cot u\left(\frac{\cot u\left(1-\csc u\right)}{\left(1+\csc u\right)\left(1-\csc u\right)}\right) =$$

$$\cot u\left(\frac{1-\csc u}{-\cot u}\right) =$$

$$\cot u\left(\frac{\cot u\left(1-\csc u\right)}{1-\csc^2 u}\right) =$$

$$\cot u\left(\frac{\csc u}{\cot u} - \frac{1}{\cot u}\right) =$$

$$\cot u\left(\frac{\frac{1}{\sin u}}{\frac{\cos u}{\sin u}}-\tan u\right)=$$

$$\cot u\left(\frac{1}{\cos u}-\tan u\right)=$$

$$\cot u(\sec u-\tan u)=\cot u(\sec u-\tan u)$$

II. Power-Reducing Identities

$$\cos^3 x=\left(1-\sin^2 x\right)\cos x$$

$$\sin^3 x=\left(1-\cos^2 x\right)\sin x$$

$$\sin^2 x=\frac{1-\cos 2x}{2}$$

$$\cos^2 x=\frac{1+\cos 2x}{2}$$

$$\tan^2 x=\frac{1-\cos 2x}{1+\cos 2x}$$

Prove: $\sin^5 x = \left(1 - 2\cos^2 x + \cos^4 x\right)\sin x$

$$= \left(1 - \cos^2 x\right)^2 \sin x$$

$$= \left(\sin^2 x\right)^2 \sin x$$

$$= \left(\sin^4 x\right)\sin x$$

$$\sin^5 x = \sin^5 x \quad \checkmark\checkmark$$