5.1 Fundamental Trig Identities

I. Basic Trig Identities

Reciprocal –

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

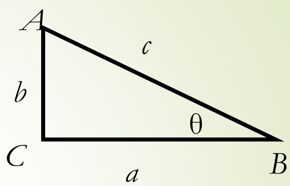
Quotient -

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

II. Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

Proof- Let θ be an angle in right triangle *ABC*.



$$a^2 + b^2 = c^2$$

$$\sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c}$$

$$\frac{a^2+b^2}{c^2} = \frac{c^2}{c^2}$$

$$\frac{a^2+b^2}{c^2} = 1$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

$$\left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2 = 1$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

$$\sin^2\theta + \cos^2\theta = 1$$

Now, divide everything by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Divide everything by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

III. Using Pythagorean Identities

Given $\cot\theta = -\frac{3}{4}$ and $\sin\theta > 0$. Use a Pythagorean Identity to find $\sin\theta$ and $\cos\theta$.

$$1+\cot^2\theta = \csc^2\theta$$

$$1+\left(-\frac{3}{4}\right)^2 = \csc^2\theta$$

$$1+\frac{9}{16} = \csc^2\theta$$

$$\frac{25}{16} = \csc^2\theta$$

$$\frac{5}{4} = \csc\theta$$

$$\sin\theta = \frac{4}{5} \qquad \cos\theta = -\frac{3}{5}$$

IV. Cofunction Identities

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

V. Even-Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

If $\cot\theta = 3$, find $\tan\left(\theta - \frac{\pi}{2}\right)$.

$$\tan\left(\theta - \frac{\pi}{2}\right) = -\tan\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\therefore \tan\left(\theta - \frac{\pi}{2}\right) = -\cot\theta = -3$$

VI. Solving Trig Equations

Find all values of x on $[0, 2\pi]$ for: $\frac{\cos^3 x}{\sin x} = \cot x$

$$\cos^3 x = \sin x\left(\frac{\cos x}{\sin x}\right)$$

$$\cos^3 x - \cos x = 0$$

$$\cos x\left(\cos^2 x - 1\right) = 0$$

$$\cos x\left(-\sin^2 x\right) = 0$$

$\cos x = 0$ or $-\sin^2 x = 0$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

EXTRANEOUS!!!

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Find all values of t on [0, 2π] for: $3\sin t = 2\cos^2 t$

$$3\sin t = 2\left(1-\sin^2 t\right)$$

$$2\sin^2 t + 3\sin t - 2 = 0$$

$$\left(2\sin t - 1\right)\left(\sin t + 2\right) = 0$$

$$\left(2\sin t - 1\right) = 0 \quad \text{or} \quad \left(\sin t + 2\right) = 0$$

$$\sin t = \frac{1}{2}$$

$$t = \frac{\pi}{6}, \frac{5\pi}{6}$$