

Notes 4.7 – Inverse Trig Functions

I. Inverse Sine Function \rightarrow Arcsin x

A.) We have to restrict the domain of $f(x) = \sin x$ to

In general: Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B.) Ex. – Find the exact values of each without a calculator.

1.) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

2.) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

3.) $\sin^{-1}\left(\sin \frac{\pi}{7}\right) = \frac{\pi}{7}$

4.) $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

II. Inverse Cosine Function $\rightarrow \text{Arccos } x$

A.) We have to restrict the domain of $f(x) = \cos x$ to

$$[0, \pi]$$

$$y = \cos x$$

In general: Domain: $[-1, 1]$

Range: $[0, \pi]$

III. Inverse Tangent Function $\rightarrow \text{Arctan } x$

A.) We have to restrict the domain of $f(x) = \tan x$ to

In general: Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

B.) Ex. – Find the exact values of each without a calculator.

$$1.) \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) \quad 2.) \tan^{-1}(\sqrt{3}) \quad 3.) \tan^{-1}\left(\sin\frac{\pi}{2}\right)$$

$$\frac{3\pi}{4}$$

$$\frac{\pi}{3}$$

$$\frac{\pi}{4}$$

IV. Composing Trig and Arc Trig Functions

A.) ALWAYS TRUE $\sin(\sin^{-1} x) = x$

$$\cos(\cos^{-1} x) = x$$

$$\tan(\tan^{-1} x) = x$$

B.) ONLY TRUE ON RESTRICTED DOMAINS

$$\sin^{-1}(\sin x) = x$$

$$\cos^{-1}(\cos x) = x$$

$$\tan^{-1}(\tan x) = x$$

C.) Given the triangle as shown with θ measured in radians, find the following.

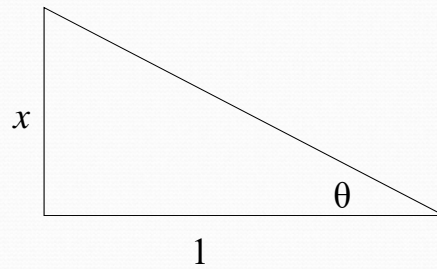
1.) $\tan \theta = x$

2.) $\tan^{-1} x = \theta$

3.) The Hypotenuse = $\sqrt{1+x^2}$

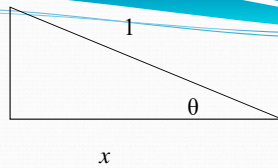
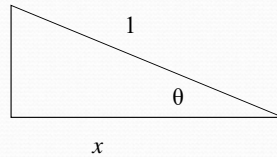
4.) $\sin(\tan^{-1} x) = \sin \theta = \frac{x}{\sqrt{1+x^2}}$

5.) $\sec(\tan^{-1} x) = \sec \theta = \sqrt{1+x^2}$



C.) Ex.- Compose each of the six basic trig functions $\cos^{-1} x$ with and reduce the composite function to an algebraic expression involving no trig functions.

First, we need to draw the triangle where $\theta = \cos^{-1} x$



$$\sin(\cos^{-1} x) = \sqrt{1-x^2}$$

$$\cos(\cos^{-1} x) = x$$

$$\tan(\cos^{-1} x) = \frac{\sqrt{1-x^2}}{x}$$

$$\cot(\cos^{-1} x) = \frac{x}{\sqrt{1-x^2}}$$

$$\sec(\cos^{-1} x) = \frac{1}{x}$$

$$\csc(\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$