4-6: GRAPHS OF COMPOSITE TRIGONOMETRIC FUNCTIONS

I. COMPOSITES

A.) Ex. - Graph the following trig functions and determine which are periodic.

1.) $f(x) = x^2 \cos x$

2.) $f(x) = (1.5 \cos x)^2$

3.) $f(x) = 2\sin(x) + x^2$

Only $f(x) = (1.5 \cos x)^2$ is periodic.

It has a period of π.

B.) Ex. - Prove the following is periodic and determine the period graphically.

$$f(x) = \sqrt{\sin^2 x}$$

$$f(x+2\pi) = \sqrt{\sin^2(x+2\pi)}$$

$$= \sqrt{(\sin(x+2\pi))^2}$$

$$= \sqrt{(\sin x)^2}$$

$$= \sqrt{\sin^2 x} = |\sin x|$$

$$= f(x)$$

$$\text{pd} = \pi$$

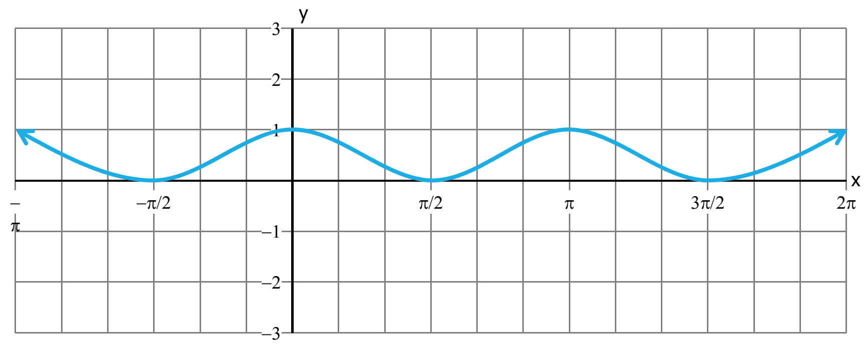
C.) Ex. – Find the domain, range, and period of each of the following functions and sketch the graphs.

1.) $f(x) = |\cos x|$ 2.) $f(x) = |\cot x|$

1.) -Domain = (-∞, ∞)

-Range = [0, 1]

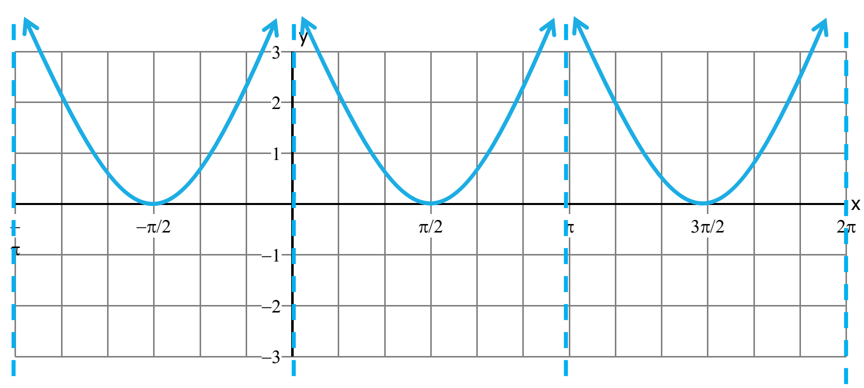
-Pd. = π



2.) -Domain = $x \neq n\pi$ for all int. n

-Range = [0, ∞)

-Pd. = π



II. ADDING A SINUSOID TO A LINEAR FUNCTION

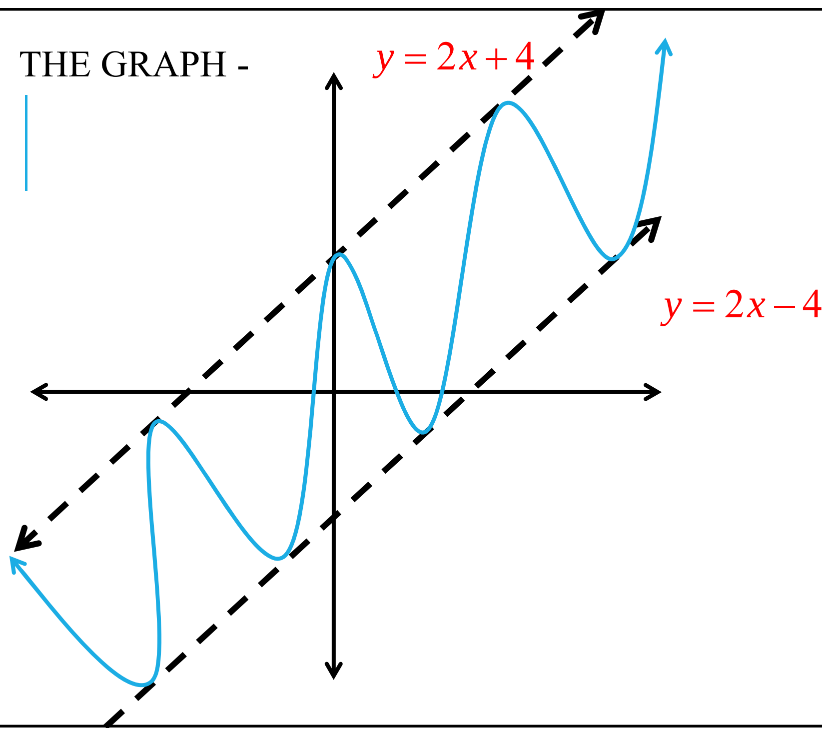
Note- NOT PERIODIC!!!

Ex- Between which two parallel lines does the following function oscillate?

$$y = 2x + 4\cos x$$

$y = 2x + 4$ and $y = 2x - 4$

THE GRAPH -



III. SUMS AND DIFFERENCES OF SINUSOIDS

A.)If $y_1 = a_1 \sin b(x - h_1)$ and $y_2 = a_2 \sin b(x - h_2)$ then

$$y_1 + y_2 = a_1 \sin b(x - h_1) + a_2 \sin b(x - h_2)$$

is a sinusoid with period $\dfrac{2\pi}{|b|}$.

i.e., They must have the SAME PERIOD!

B.) Ex- Let $f(x) = 2\cos 3x + 3\sin 3x$

1.) Find the period.

2.) Estimate the amplitude and phase shift to the nearest thousandth.

3.) Give a sinusoid that approximates $f(x)$.

use your TI-83

$$\text{pd} = \frac{2\pi}{|3|} = \frac{2\pi}{3}$$

$a \approx 3.606$

ps ≈ -0.196

$$y \approx 3.606 \sin\left(3(x + .196)\right)$$

C.) Ex- Let $f(x) = \sin\frac{2}{5}x - 2\cos\frac{2}{5}x$

1.) Find the period.

2.) Estimate the amplitude and phase shift to the nearest thousandth.

3.) Give a sinusoid that approximates $f(x)$.

$$\text{pd} = \frac{2\pi}{\left|\frac{2}{5}\right|} = 5\pi$$

$$a \approx 2.236$$

$$\text{ps} \approx 2.768$$

$$y \approx 2.236\sin\left(\frac{2}{5}(x - 2.768)\right)$$

D.) Ex.- Show that $f(x) = \cos 5x + \sin 3x$ is not a sinusoid, but it is periodic.

Not a sinusoid- $\cos 5x$ pd. $= \frac{2\pi}{5}$

$\sin 3x$ pd. $= \frac{2\pi}{3}$

Periodic -

$$f(x+2\pi) = \cos 5(x+2\pi) + \sin 3(x+2\pi)$$

$$= \cos(5x+10\pi) + \sin(3x+6\pi)$$

$$= \cos(5x) + \sin(3x)$$

$$= f(x)$$

IV. DAMPED OSCILLATION

A.) Both the $\sin(bt)$ and the $\cos(bt)$ oscillate between the values of -1 and 1. It should make sense that any function $f(x)$ we multiply the sin or cos function by would then oscillate between - $f(x)$ and $f(x)$. If $f(x)$ reduces the amplitude of the wave, $f(x)$ creates a DAMPED OSCILLATION where $f(x)$ is called the DAMPING FACTOR.

B.) Ex.- Determine the damping factor and where the damping occurs in each of the following.

	D.F. –	Damping occurs
1.) $y = 2^x \cos x$	$y = 2^x$	$x \to -\infty$
2.) $y = 2^{-x} \sin x$	$y = 2^{-x}$	$x \to \infty$
3.) $y = -2x \cos x$	$y = -2x$	$x \to 0$