4.4-GRAPHS OF SINE AND COSINE: SINUSOIDS

I. COMBINING PHASE SHIFTS AND PERIOD CHANGE

Construct a sinusoid with a period of $\frac{\pi}{5}$ and amplitude of 6 that passes through (2,0).

$$y = a\sin\left(bx + c\right) + d$$

We need to determine *a*, *b*, *c*, and *d*.

$|a| = 6$ $\quad$ $pd = \frac{2\pi}{|b|}$ $\quad$ $\frac{\pi}{5} = \frac{2\pi}{|b|}$ $\quad$ $|b| = 10$

Now for *c*…

The sine curve goes through the origin at (0, 0). This curve passes through the point (2, 0), therefore, it is a horizontal translation 2 units to the right. This gives us a possible sinusoid of $f(x) = 6\sin\left(10\left(x-2\right)\right)$ or $f(x) = -6\sin\left(10\left(x-2\right)\right)$

Construct a sinusoid with a minimum value of y = -5 at x = 0 and a maximum value of y = 25 at x = 32.

Hint: First choose which general type of sinusoid you are constructing. (I would probably choose the cosine function for this one!)

$$y = a\cos\left(bx + c\right) + d$$

We need to determine a, b, c, and d.

$$|a| = \frac{25-(-5)}{2}$$

$$|a| = 15$$

$$d = \frac{\max + \min}{2}$$

$$d = 10$$

$$pd = \frac{2\pi}{|b|}$$

$$64 = \frac{2\pi}{|b|}$$

$$|b| = \frac{\pi}{32}$$

Since the sinusoid passes through its minimum value at x = 0, a would be negative, giving us a sinusoid of

$$y = -15\cos\left(\frac{\pi}{32}x\right) + 10$$

II. CONSTRUCTING A SINUSOIDAL MODEL USING TIME

$$f(t) = A\sin\left(B(t-T)\right) + C$$

1.) Determine the amplitude. $A = \frac{\text{Max-Min}}{2}$

2.) Determine the vert. shift. $C = \frac{\text{Max+Min}}{2}$

3.) Determine the period. $B = \frac{2\pi}{\text{pd}}$

4.) Choose the appropriate model

based on the behavior at the given time.

Max at *t*: $f(t) = A\cos\left(B(t-T)\right)+C$

Min at *t*: $f(t) = -A\cos\left(B(t-T)\right)+C$

½ way between Min and Max at *t*:

$$f(t) = A\sin\left(B(t-T)\right)+C$$

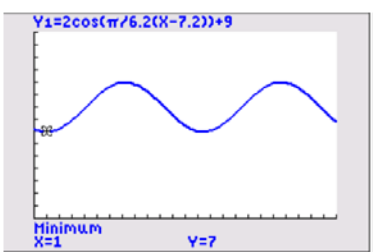
½ way between Max and Min at *t*:

$$f(t) = -A\sin\left(B(t-T)\right)+C$$

On Sept. 4, 2016 the high tide at the Jersey shore occurs at 7:12 AM. At that time the water at the Atlantic City Pier is measured to be 11 feet deep. AT 1:24 PM low tide occurs and the water measures only 7 feet deep. Assume the depth of the water is a sinusoidal function of time with a period of ½ a lunar day (12 hr. 24 min.)

A.) At what time Sept. 4, 2016 does the first low tide occur?

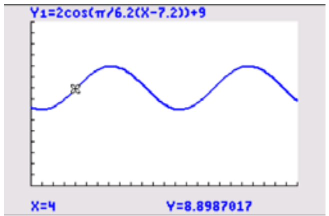
$$A = \frac{11-7}{2} = 2 \quad B = \frac{2\pi}{12.4} = \frac{\pi}{6.2} \quad C = \frac{11+7}{2} = 9 \quad T = 7.2$$

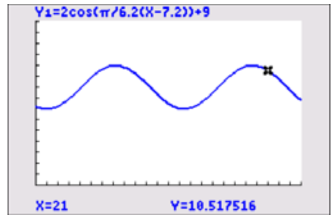


$$f(t) = 2\cos\left(\frac{\pi}{6.2}(t-7.2)\right)+9$$

$t = 1:00$ am

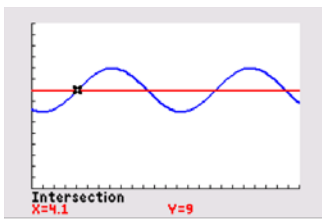
B.) What is the approximate depth of the water at 4:00 AM and 9:00 PM?





$f(4) \approx 8.899$ ft $\qquad f(21) \approx 10.518$ ft

C.) What is the first time on Sept. 4, 2016 that the water was 9 feet deep?



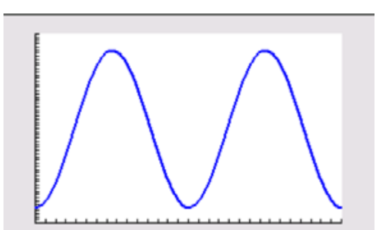
$t = 4:06$ am

Joey and Sally are riding on a Ferris wheel at the county fair. The Ferris wheel has a diameter of 50 feet and turns at 4 revolutions per minute with its lowest point 5 feet above the ground. Assume that Joey and Sally's height h above the ground is a function of time t in seconds, where $t = 0$ represents the lowest point of the wheel.

A)Write an equation for h.

$A = \frac{55-5}{2} = 25 \quad B = \frac{2\pi}{15} \quad C = \frac{55+5}{2} = 30 \qquad h(t) = -25\cos\left(\frac{2\pi}{15}t\right) + 30$

B.) Draw a graph of h for $0 \le t \le 30$.



C.) Use h to estimate Joey and Sally's height at $t = 8$ seconds. $h(8) \approx 54.454$ ft