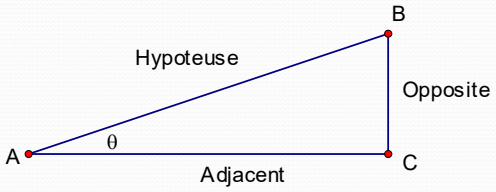
4.2: Trigonometric Functions of Acute Angles

I. Right Triangle Trigonometry

A. Definition: Trigonometric Functions

Let *theta* be an acute angle in the right triangle *ABC*, Then:

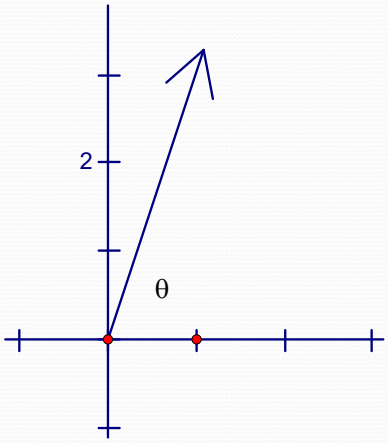


$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

B. Standard Position

An angle is in standard position if it has one ray along the positive *x-axis*.



II. Two Famous Triangles

A. Find the values of all six trigonometric functions for an acute angle of 45 degrees.

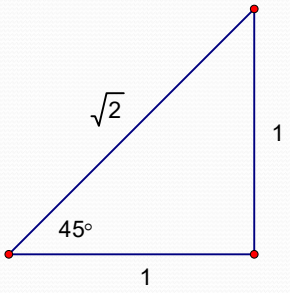
1. $\sin 45^\circ = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.707...$

2. $\cos 45^\circ = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.707...$

3. $\tan 45^\circ = \frac{opp}{adj} = \frac{1}{1} = 1$

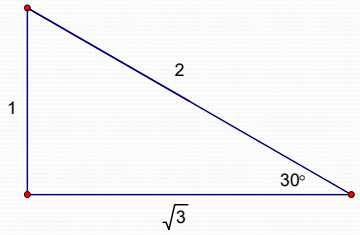
4. $\csc 45^\circ = \frac{hyp}{opp} = \sqrt{2} \approx 1.414...$

5. $\sec 45^\circ = \frac{hyp}{adj} = \sqrt{2} \approx 1.414...$



6. $\cot 45^\circ = \frac{adj}{opp} = \frac{1}{1} = 1$

B. Find the values of all six trigonometric functions for an acute angle of 30 degrees.



1. $\sin 30^\circ = \frac{opp}{hyp} = \frac{1}{2}$

2. $\cos 30^\circ = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} \approx 0.866...$

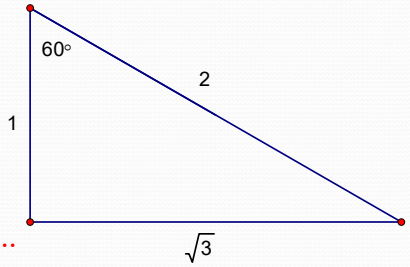
3. $\tan 30^\circ = \frac{opp}{adj} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.577...$

4. $\csc 30^\circ = \frac{hyp}{opp} = \frac{2}{1} = 2$

5. $\sec 30^\circ = \frac{hyp}{adj} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \approx 1.155...$

6. $\cot 30^\circ = \frac{adj}{opp} = \frac{\sqrt{3}}{1} = \sqrt{3} \approx 1.732...$

C. Find the values of all six trigonometric functions for an acute angle of 60 degrees.



1. $\sin 60^\circ = \frac{opp}{hyp} = \frac{\sqrt{3}}{2} \approx 0.866...$

2. $\cos 60^\circ = \frac{adj}{hyp} = \frac{1}{2}$

3. $\tan 60^\circ = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \sqrt{3} \approx 1.732...$

4. $\csc 60^\circ = \frac{hyp}{opp} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \approx 1.155...$

5. $\sec 60^\circ = \frac{hyp}{adj} = \frac{2}{1} = 2$

6. $\cot 60^\circ = \frac{adj}{opp} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.577...$

II. Exact Values

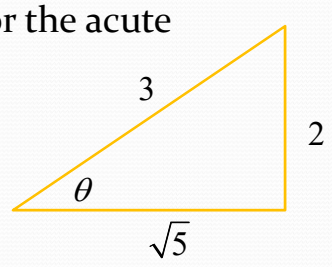
MEMORIZE!!!

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\varnothing$

III. Finding Trig Values

Ex.- Find the other 5 trig ratios for the acute angle θ given the fact the

$$\sin\theta = \frac{2}{3}$$



$$\cos\theta = \frac{\sqrt{5}}{3} \quad \tan\theta = \frac{2\sqrt{5}}{5} \quad \cot\theta = \frac{\sqrt{5}}{2}$$

$$\sec\theta = \frac{3\sqrt{5}}{5} \quad \csc\theta = \frac{3}{2}$$

IV. Inverse Trig Functions

$\sin^{-1} x,\ \cos^{-1} x,\ \text{and}\ \tan^{-1} x$

are all inverse trigonometric function, they ARE NOT RECIPROCALS!!!!

Ex- Evaluate the following inverse trig functions for the acute angle in radians without a calculator.

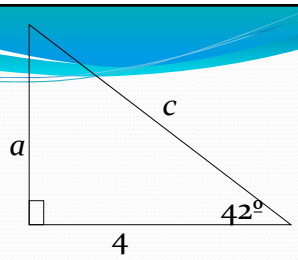
A.) $\sin^{-1} \dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}$ B.) $\tan^{-1} 1 = \dfrac{\pi}{4}$

Ex- Find a and c.

$\cos 42° = \dfrac{4}{c}$ $\tan 42° = \dfrac{a}{4}$

$c(\cos 42°) = 4$ $a = 4(\tan 42°)$

$c \approx 5.383$ $a \approx 3.602$

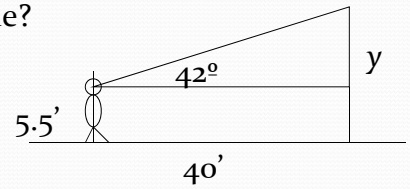


Ex- The angle of elevation to the top of a flagpole is 42 when Joey is 40 feet from the base of the flagpole. If Joey is measuring the angle with a protractor , straw, and a piece of string and his eyes are at a height of 5 feet 6 inches from the ground, how high is the flagpole?

$\tan 42° = \dfrac{y}{40}$

$y = 40(\tan 42°)$

$y \approx 36.02$



$\text{flagpole} = 36.02 + 5.5$

$= 41.52$