4.1: Angles and Their Measures

I. Degrees and Radians

A. A degree is a unit of angular measure equal to $1/180^{th}$ of a straight angle.

B. A degree is broken up into minutes and seconds (in the DMS—degree-minute-second system) as follows:

1. 60 minutes equal 1 degree
2. 60 seconds equal 1 minute
3. Also, 3600 seconds equal 1 degree

C. Example 1: working with DMS measure

1. Approximate the angle 13°32'50" as a decimal to the nearest 10,000th of a degree.

$$13^\circ 32'50'' = 13^\circ + \left(\frac{32}{60}\right)^\circ + \left(\frac{50}{3600}\right)^\circ \approx \boxed{13.5472^\circ}$$

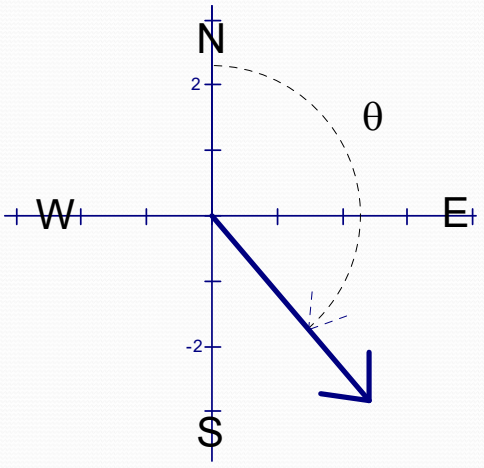
2. Approximate the angle 32.519° in terms of degrees, minutes, and seconds.

$$32.519^\circ = 32^\circ + 60(0.519)^\circ = 32^\circ + 31.14^\circ$$

$$= 32^\circ 31' + 60(0.14)'$$

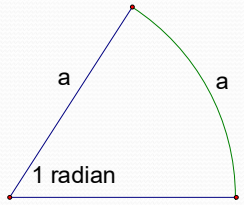
$$= \boxed{32^\circ 31'8.4''}$$

D. In navigation, the *course* or *bearing* of an object is sometimes given as the angle of the *line of travel* measured clockwise from due north.



E. Radians

1. A central angle of a circle has measure 1 *radian* if it intercepts an arc with the same length as the radius.



2. Conversion between degrees and radians

$$360^\circ = 2\pi \text{ radians}$$

$$1^\circ = \frac{\pi}{180} \text{ radians}$$

3. Example 2:

a. How many radians are in 72 degrees?

$$72^\circ\left(\frac{\pi}{180}\right) = \frac{2\pi}{5} radians$$

b. How many degrees are in $\pi/5$ Radians?

$$\frac{\pi}{5}\left(\frac{180}{\pi}\right) = 36^\circ$$

c. Find the length of an arc intercepted by a central angle of 3 radians in a circle of radius 6 inches.

$$Circumference = 2\pi \cdot 6 = 12\pi in$$

$$Portion\ of\ circle = \frac{3}{2\pi}$$

$$arc = 12\pi\left(\frac{3}{2\pi}\right) = 18in$$

II. Circular Arc Length

A. Arc length formulas: if *theta* is a central angle in a circle of radius *r*, the following formulas will give the arc length *s*,

Radians: $s = r\theta$

Degrees: $s = \frac{\pi r \theta}{180}$

B. Example 3:

1. Suppose that an angle inscribed within a circle of radius 4 centimeters has measure 2.3 radians. Determine the length of the arc defined by the angle.

$$s = 4 \cdot 2.3 = \boxed{9.2cm}$$

2. Example 4: Suppose that a central angle of a circle of radius 12 meters intercepts an arc of length 14 meters. Find the radian measure of the angle.

$$14 = 12\theta$$

$$\boxed{\frac{7}{6} radians = \theta}$$

C. Example 4: A person is seated on the end of a see-saw whose total length is 5m. The see-saw moves up and down through a 28 degree angle every 3 seconds. Through what distance does the person move in a minute?

$$s = \frac{\pi \cdot 2.5 \cdot 56}{180} \approx 2.44m$$

$$\frac{2.44m}{3\sec} = \frac{48.86m}{60\sec}$$

$$\boxed{48.86meters}$$

III. Angular and Linear Motion

A. We can measure speed in both linear (such as miles per hour) and angular speed (such as revolutions per minute).

B. Example 5: Suppose that the wheels on a tractor have a diameter of 40 inches and that the angular speed of the tires is 550 rotations per minute. What is the truck's speed in miles per hour?

$$\frac{550rot}{\min} \cdot \frac{60\min}{hr} \cdot \frac{2\pi radians}{rot} \cdot \frac{1ft}{12in} \cdot \frac{20in}{1radian} \cdot \frac{1mi}{5280ft}$$

$$= \boxed{65.45\frac{mi}{hr}}$$

C. Nautical Mile

1. A *nautical mile* is the length of 1 minute of arc (1/60 of a degree) length along the earth's equator.
2. The familiar "land mile" is called the *statute mile*.
3. We find the conversion below (the radius of the earth is approximately 3956 stat miles):

$$1' = \left(\frac{1}{60}\right)^{\circ} \cdot \left(\frac{\pi rad}{180^{\circ}}\right) = \frac{\pi}{10,800} radians$$

$$1 \text{ naut mile } = 3956 \cdot \left(\frac{\pi}{10,800}\right) \text{ stat mi } \approx 1.15 \text{ stat mi}$$

$$1 \text{ stat mi} = \frac{10,800}{3956\pi} \text{naut mi} \approx 0.87 \text{ naut mi}$$

distance conversions

1 stat mile $\approx$ 0.87 naut mile

1 naut mile $\approx$ 1.15 statute mile

3. Example 6: A pilot flies from New Jersey to Florida, a distance of approximately 954 miles. How many nautical miles is it from New Jersey to Florida.

$$954 \text{ stat mi} \approx \frac{954 \cdot 10,800}{3956\pi} \approx \boxed{829 \text{ naut mi.}}$$