NOTES 3.6 – MATHEMATICS OF FINANCE

I. SIMPLE INTEREST

$$I = Prt$$

$$A = P(1 + rt)$$

Where P = Principal, r = rate, t = time in years, and A =total amount

II. COMPOUND INTEREST

$$A = P\left(1 + \frac{r}{k}\right)^{kt}$$

Where P = Principal, r = rate, t = time in years, k = # of times compounded per year and A =total amount

III. EXAMPLES

A.) SPSE you invest $1,000.00 at 6% compounded monthly. How much will your investment be worth in 5 years?

$$A = 1000\left(1 + \frac{.06}{12}\right)^{12(5)} = \$1{,}348.85$$

B.) SPSE you invest $500.00 at 10% compounded daily. How long will it take to reach $2,000.00?

$$2000 = 500\left(1+\frac{.1}{365}\right)^{365t}$$

$$4 = \left(1+\frac{.1}{365}\right)^{365t}$$

$$4 = 1.000273973^{365t}$$

$$\ln 4 = \ln 1.000273973^{365t} \qquad t \approx 13.864 \text{ yrs}$$

C.) What rate will guarantee doubling your money in ten years if it is to compounded quarterly?

$$2 = 1\left(1+\frac{r}{4}\right)^{4(10)}$$

$$2 = \left(1+\frac{r}{4}\right)^{40} \qquad r = .0699187 \approx 7\%$$

$$1.017479692 = 1+\frac{r}{4}$$

$$.017479692 = \frac{r}{4}$$

IV. COMPOUNDING CONTINUOUSLY

$$A = Pe^{rt}$$

A.) How much would your investment at the end of 10 years if you deposit $10,000.00 in a certificate of deposit compounded continuously at 7.5% interest?

$$A = 10000e^{.075(10)}$$

$$A = \$21,170.00$$

V. ANNUAL PERCENTAGE YIELD

Basis for comparing investments

Amount of interest $1 earns in 1 year.

$$APY = \left(1 + \frac{r}{k}\right)^{k} - 1$$

Which is more attractive; 9.75% compounded quarterly or 9.7% compounded monthly?

$$AY_1 = \left(1 + \frac{.0975}{4}\right)^{4} - 1 \qquad AY_2 = \left(1 + \frac{.097}{12}\right)^{12} - 1$$

$$AY_1 = .101123 \qquad AY_2 = .1014308$$

VI. ANNUITIES-FUTURE VALUE

A sequence of equal periodic payments ordinarily made at the end of each period, the same time the interest is being deposited

Ex: SPSE you put $200 into a retirement fund which pays 10% compounded quarterly. How much will be in the account at the end of 1 year?

$$Q_1 = 200$$

$$Q_2 = 200 + 200(1.025)$$

$$Q_3 = 200 + 200(1.025) + 200(1.025)^2$$

$$Q_4 = 200 + 200(1.025) + 200(1.025)^2 + 200(1.025)^3$$

$$Q_4 = \$830.50$$

The Future Value of an Annuity consisting of n equal periodic payments of R dollars at an interest rate i per compounding period can be given by the equation

$$FV = R\frac{(1+i)^n - 1}{i}$$

Ex. – What is the future value of an annuity that consist of $200 monthly contributions earning 10% annual interest after 20 years?

$$FV = 200\frac{\left(1+\frac{.1}{12}\right)^{20(12)} - 1}{\frac{.1}{12}} = \$151,873.76$$

VII. LOANS AND MORTGAGES

$$PV = R\left(\frac{1-(1+i)^{-n}}{i}\right)$$

Is the Present Value of an annuity of n equal payments of R dollars earning an interest rate I per period.

Ex. – Joe purchases a new truck for $28,000 using a $2,000 down payment and making out a 5-year loan at 2.9% interest. What are the monthly payments?

$$26000 = R\left(\frac{1-(1+\frac{.029}{12})^{-60}}{\frac{.029}{12}}\right)$$

$$26000 = 55.79022549R$$

$$R = \$466.04$$