3-5 HOMEWORK #2

p. 301 #30, 33, 36, 38, 39, 41, 44, 47, 53

30. Multiply both sides by $2 \cdot 2^x$, leaving $(2^x)^2 + 1 = 6 \cdot 2^x$, or $(2^x)^2 - 6 \cdot 2^x + 1 = 0$. This is quadratic in 2^x, leading to $2^x = \dfrac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}$. Then $x = \dfrac{\ln(3 \pm 2\sqrt{2})}{\ln 2}$ $= \log_2(3 \pm 2\sqrt{2}) \approx \pm 2.5431$.

33. $\dfrac{500}{200} = 1 + 25e^{0.3x}$, so $e^{0.3x} = \dfrac{3}{50} = 0.06$, and therefore $x = \dfrac{1}{0.3} \ln 0.06 \approx -9.3780$.

36. Multiply by 2, then combine the logarithms to obtain $\log \dfrac{x^2}{x + 4} = 2$. Then $\dfrac{x^2}{x + 4} = 10^2 = 100$, so $x^2 = 100(x + 4)$. The solutions to this quadratic equation are $x = \dfrac{100 \pm \sqrt{10000 + 1600}}{2} = 50 \pm 10\sqrt{29}$. The original equation requires that $x > 0$, so $50 - 10\sqrt{29}$ is extraneous; the only actual solution is $x = 50 + 10\sqrt{29} \approx 103.8517$.

38. $\log[(x - 2)(x + 5)] = 2 \log 3$, so $(x - 2)(x + 5) = 9$, or $x^2 + 3x - 19 = 0$.
Then $x = \dfrac{-3 \pm \sqrt{9 + 76}}{2} = -\dfrac{3}{2} \pm \dfrac{1}{2}\sqrt{85}$. The actual solution is $x = -\dfrac{3}{2} + \dfrac{1}{2}\sqrt{85} \approx 3.1098$; since $x - 2$ must be positive, the other algebraic solution, $x = -\dfrac{3}{2} - \dfrac{1}{2}\sqrt{85}$, is extraneous.

39. A \$100 bill has the value of 1000, or 10^3, dimes so they differ by an order of magnitude of 3.

41. $7 - 5.5 = 1.5$. They differ by an order of magnitude of 1.5.

44. Given

$$\beta_1 = 10 \log \frac{I_1}{I_0} = 70$$

$$\beta_2 = 10 \log \frac{I_2}{I_0} = 10,$$

we seek the logarithm of the ratio I_1/I_2.

$$10 \log \frac{I_1}{I_0} - 10 \log \frac{I_2}{I_0} = \beta_1 - \beta_2$$

$$10\left(\log \frac{I_1}{I_0} - \log \frac{I_2}{I_0}\right) = 70 - 10$$

$$10 \log \frac{I_1}{I_2} = 60$$

$$\log \frac{I_1}{I_2} = 6$$

The two intensities differ by 6 orders of magnitude.

47. (a) Carbonated water: $-\log [H^+] = 3.9$

$$\log [H^+] = -3.9$$

$$[H^+] = 10^{-3.9} \approx 1.26 \times 10^{-4}$$

Household ammonia: $-\log [H^+] = 11.9$

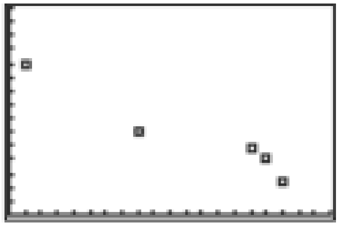
$$\log [H^+] = -11.9$$

$$[H^+] = 10^{-11.9} \approx 1.26 \times 10^{-12}$$

(b) $$\frac{[H^+] \text{ of carbonated water}}{[H^+] \text{ of household ammonia}} = \frac{10^{-3.9}}{10^{-11.9}} = 10^8$$

(c) They differ by an order of magnitude of 8.

53. (a)



[0, 20] by [0, 15]

(b) The scatter plot is better because it accurately represents the times between the measurements. The equal spacing on the bar graph suggests that the measurements were taken at equally spaced intervals, which distorts our perceptions of how the consumption has changed over time.