NOTES 3.5 – EQUATION SOLVING AND MODELING

I. ONE-TO-ONE PROPERTIES

$$\text{if } b^a = b^c \text{ then } a = c$$

$$\text{if } \log_b a = \log_b c \text{ then } a = c$$

II. EXAMPLES

A.) $18\left(\frac{1}{2}\right)^{\frac{x}{2}} = 4\frac{1}{2}$

$$\left(\frac{1}{2}\right)^{\frac{x}{2}} = \frac{1}{4}$$

$$(2)^{-\frac{x}{2}} = 2^{-2}$$

$$-\frac{x}{2} = -2$$

$$\frac{x}{2} = 2$$

$$x = 4$$

B.) $e^x - 5e^{-x} = 4$

$$e^x - \frac{5}{e^x} = 4$$

$$e^x\left(e^x - \frac{5}{e^x} = 4\right)$$

$$e^{2x} - 5 = 4e^x$$

$$e^{2x} - 4e^x - 5 = 0$$

$$e^{2x} - 4e^x - 5 = 0$$

$$\left(e^x - 5\right)\left(e^x + 1\right) = 0$$

$$e^x - 5 = 0 \text{ or } e^x + 1 = 0$$

$$e^x = 5$$

$$x = \ln 5$$

III. SOLVING LOGARITHMIC EQUATIONS

****DEFINE THE DOMAIN FIRST!!!****

Can use properties, but be careful!!!

$$2\log_3 x + 3 = 4 \text{ vs. } \log_3 x^2 + 3 = 4$$

$2\log_3 x + 3 = 4$	$\log_3 x^2 + 3 = 4$
$2\log_3 x = 1$	$\log_3 x^2 = 1$
$\log_3 x = \frac{1}{2}$	$x^2 = 3$
$x = 3^{\frac{1}{2}}$	$x = \pm\sqrt{3}$
$x = \sqrt{3}$	

Support Graphically!!!!

$$\ln\left(3x-2\right)+\ln(x-1)=2\ln x$$

$$\ln\left((3x-2)(x-1)\right)=\ln x^2$$

$$\ln\left(3x^2-5x+2\right)=\ln x^2$$

$$\left(3x^2-5x+2\right)=x^2$$

$$2x^2-5x+2=0$$

$$x=2 \text{ but } x\neq\frac{1}{2}$$

IV. ORDERS OF MAGNITUDE

A.) Def. – The common logarithm of a positive quantity is known as an ORDER OF MAGNITUDE. It is used to compare like quantities.

B.) Ex. – A kilometer is 3 orders of magnitude greater than a meter.

-$10,000.00 is 6 orders of magnitude greater than a penny.

V. RICHTER SCALE

A.) $R = \log \frac{a}{T} + B$

a = ampl. in micrometers of vert. ground move.

T = pd. of assoc. seismic wave in seconds.

B = weakening of seismic wave w/ inc. in dist. from epicenter.

B.) Ex. – How many times more severe is an earthquake measuring 7.9 on the Richter scale than one measuring 5.9?

$$R_1 = 7.9 = \log \frac{a_1}{T} + B$$

$$R_2 = 5.9 = \log \frac{a_2}{T} + B$$

$$R_1 - R_2 = \left(\log \frac{a_1}{T} + B \right) - \left(\log \frac{a_2}{T} + B \right)$$

$$7.9 - 5.9 = \log \frac{a_1}{T} - \log \frac{a_2}{T}$$

$$2 = \log \left(\frac{\frac{a_1}{T}}{\frac{a_2}{T}} \right)$$

$$2 = \log \left(\frac{a_1}{a_2} \right)$$

$$10^2 = \frac{a_1}{a_2}$$

100 times greater

VI. CHEMICAL ACIDITY

A.) Acidity is measured by the concentration of hydrogen ions in the solution. (in moles/liter)

$$H^{+} \rightarrow \text{ Hydrogen Concentration}$$

$$pH \rightarrow \text{ measure of acidity} = -\log H^{+}$$

$$\text{Note: This is the opposite}$$
$$\text{of the common log } H^{+}$$

B.) Ex. – A sample of vinegar has a *pH* of 3.2, while a sample of bottled water has a *pH* of 6.2.

1.) Find the hydrogen concentration of each.

2.) Determine how many times greater the hydrogen-ion concentration of the vinegar is than that of the water.

3.) Find the order of magnitude.

1.) $-\log H^{+}_{(\text{vinegar})} = 3.2$

$H^{+}_{(\text{vinegar})} = 10^{-3.2} \approx 6.310 \times 10^{-4}$ moles/liter

$-\log H^{+}_{(\text{water})} = 6.2$

$H^{+}_{(\text{water})} = 10^{-6.2} \approx 6.310 \times 10^{-7}$ moles/liter

2.) $\dfrac{H^{+}_{(\text{vinegar})}}{H^{+}_{(\text{water})}} = \dfrac{10^{-3.2}}{10^{-6.2}} = 10^{3}$

The hydrogen-ion concentration of the vinegar is 10^{3} or 1000 times greater than that of the bottled water.

3.) Which means it is 3 orders of magnitude greater.

VII. NEWTON'S LAW OF COOLING

A.) The rate at which an object's temperature is changing at any given time is roughly proportional to the difference between its temperature and the temperature of the surrounding medium.

$$T = T_m + \left(T_0 - T_m\right)e^{-kt}$$

T_m = surrounding temp.

T_0 = initial temp.

B.) Ex- A pie is taken from the oven with an initial internal temperature of 98º C and placed in a room where the temperature is 25º C. If ten minutes later, the temperature of the pie is 82º C, determine when the pie's internal temperature is 40º C.

$$T = 25 + (98 - 25)e^{-kt}$$

$$T = 25 + (73)e^{-kt}$$

$$82 = 25 + (73)e^{-10k}$$

$$57 = 73e^{-10k}$$

$$\frac{57}{73} = e^{-10k}$$

$$\ln\left(\frac{57}{73}\right) = \ln\left(e^{-10k}\right)$$

$$\ln\left(\frac{57}{73}\right) = -10k$$

$$k = \frac{\ln\left(\frac{57}{73}\right)}{-10} \approx .0247$$

$$T = 25 + 73e^{-.0247t}$$

$$40 = 25 + 73e^{-.0247t}$$

$$15 = 73e^{-.0247t}$$

$$\frac{15}{73} = e^{-.0247t}$$

$$\ln\left(\frac{15}{73}\right) = \ln\left(e^{-.0247t}\right)$$

$$\ln\left(\frac{15}{73}\right) = -.0247t$$

$$t = \frac{\ln\left(\frac{15}{73}\right)}{-.0247} \approx 64 \text{ min.}$$