

NOTES 3.4 – PROPERTIES OF LOGARITHMIC FUNCTIONS

I. PROPERTIES

Let b , R , and S be real numbers with $b \neq 1$, and c be any real number.

1.) Product Rule- $\log_b (RS) = \log_b R + \log_b S$

2.) Quotient Rule- $\log_b \left(\frac{R}{S} \right) = \log_b R - \log_b S$

3.) Power Rule- $\log_b R^c = c \log_b R$

II. EXAMPLES

Use the properties of logs to rewrite the expression as a sum and/or difference of logs.

$$\text{A.) } \log \frac{x^3 y^4}{\sqrt{z}} = 3 \log x + 4 \log y - \frac{1}{2} \log z$$

$$\text{B.) } \log_5 125 \sqrt[3]{x^2 y} = 3 + \frac{2}{3} \log_5 x + \frac{1}{3} \log_5 y$$

Use the properties of logs to rewrite each expression as a single logarithm.

$$\text{A.) } 5 \ln x - 3 \ln y = \ln \left(\frac{x^5}{y^3} \right)$$

$$\text{B.) } 5 + 3 \log_2 x - \frac{2}{3} \log_2 y = \log_2 \left(\frac{32x^3}{\sqrt[3]{y^2}} \right)$$

III. CHANGE OF BASE PROPERTY

For positive real numbers a , b , and x with $a \neq 1$
and $b \neq 1$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

IV. EXAMPLES

Use the change of base property to evaluate the
following logs.

Using Calculator!

$$\text{A.) } \log_3 10 = \frac{\log 10}{\log 3} \approx \frac{1}{.477} \approx 2.096$$

$$\text{B.) } \log_{\frac{1}{2}} 8 = \frac{\ln 8}{\ln \frac{1}{2}} = \frac{\ln 8}{\ln 1 - \ln 2} = \frac{\ln 8}{-\ln 2} = -3$$

V. APPLICATION

Describe how to transform the graph of $f(x) = \ln x$ into the graph of the given function.

$$\text{A.) } \log_3 x = \frac{\ln x}{\ln 3} = \frac{1}{\ln 3} \ln x$$

-a vertical shrink by a factor of $\frac{1}{\ln 3}$.

$$\text{B.) } \log_{\frac{1}{3}}(x-2)$$

$$= \frac{\ln(x-2)}{\ln \frac{1}{3}} = \frac{\ln(x-2)}{\ln 1 - \ln 3} = -\frac{1}{\ln 3} \ln(x-2)$$

-ref. over x , vert. shrink by $\frac{1}{\ln 3}$, trans 2 right.

VI. RE-EXPRESSING DATA

Activity 3.4.