NOTES 3.3 – LOGARITHMIC FUNCTIONS

I. INVERSES OF EXPONENTIAL FUNCTIONS

$$y = ab^x \qquad f(x) = ab^x$$

$$x = ab^y \qquad f^{-1}(x) = \log_b \frac{x}{a}$$

Thm: if $x > 0$ and $0 < b \neq 1$, then

$$y = \log_b x \Leftrightarrow b^y = x$$

II. EXAMPLES

A.) $\log_2 8 \quad = 3$

B.) $\log_3 3\sqrt{3} \quad = \frac{3}{2}$

C.) $\log_8 1 \quad = 0$

III. PROPERTIES OF LOGS

A.) $\log_b 1 = 0 \quad \rightarrow \quad b^0 = 1$

B.) $\log_b b = 1 \quad \rightarrow \quad b^1 = b$

C.) $\log_b b^y = y \quad \rightarrow \quad b^y = b^y$

D.) $b^{\log_b x} = x \quad \rightarrow \quad \log_b x = \log_b x$

IV. COMMON LOGS- BASE 10

A.) $\log 1000 \quad = 3$

B.) $\log \sqrt[3]{10} \quad = \frac{1}{3}$

C.) $\log \frac{1}{100} \quad = -2$

V. SOLVING SIMPLE LOG. EQ.

A.) $\log\ x = 5$ $\qquad x = 100,000$

B.) $\log_3 \frac{1}{81} = x$ $\qquad x = -4$

C.) $\log_x 49 = 2$ $\qquad x = 7$

VI. THE NATURAL LOG - BASE *E*

$$y = \ln\ x \iff e^y = x$$

VII. EXAMPLES

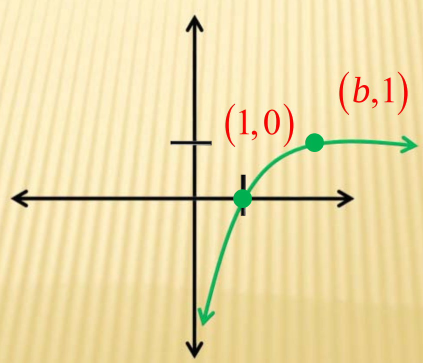
A.) $\ln \sqrt{e} \quad = \frac{1}{2}$

B.) $\ln e^5 \quad = 5$

C.) $e^{\ln 6} \quad = 6$

IX. GRAPHING LOGARITHMIC FUNCTIONS

$$y = \log_b x$$



X. TRANSFORMING LOGS

Describe how to transform the graph of $y = \ln x$ or $y = \log_b x$ to the following functions.

A.) $g(x) = \ln 2x$ - Hor. shrink by ½

B.) $h(x) = \ln(5 - x)$ - Ref. over *y*, trans 5 right

C.) $j(x) = 2\log_3 x$ - Vert. stretch by 2

XI. APPLICATIONS

Sound Decibels- Def: The level of sound of intensity in decibels (dB) is

$$B = 10\log\left(\frac{I}{I_0}\right)$$

Where *B* is the number of decibels, *I* is the sound intensity in watts/sq.m., and $I_0 = 10^{-12}$ w/sq.m. is the threshold of human hearing

Ex. Determine the decibel level for city traffic given that the intensity is 10^{-5}.

$$B = 10\log\frac{10^{-5}}{10^{-12}}$$

$$B = 10\log 10^{7}$$

$$B = 70\ \text{dB}$$