NOTES 3.1 – EXPONENTIAL AND LOGISTIC FUNCTIONS

I. DEFINITION:

Let *a* and *b* be real numbers. An EXPONENTIAL FUNCTION in *x* is a function with that can be written in the form

$f(x) = ab^x$, where $a \neq 0$ and *b* is a positive number not equal to 1. *a* is the initial value and *b* is the base.

II. $f(x) = ab^x$

- If… then…

$x = 0 \quad f(x) = a$

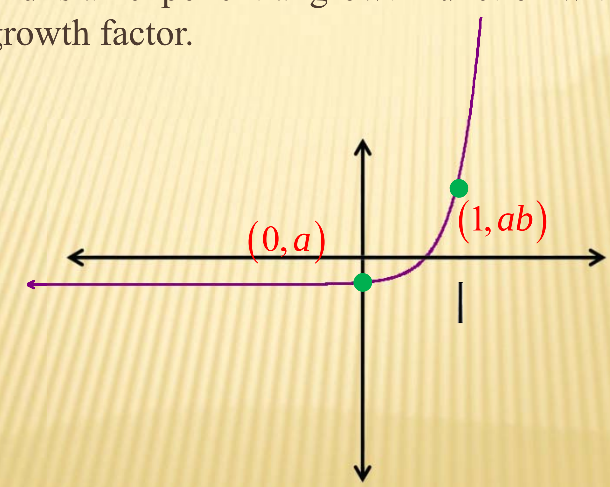
$x = \frac{m}{n} \quad f(x) = ab^{\frac{m}{n}} = a(\sqrt[n]{b})^m = a(\sqrt[n]{b^m})$

$x = -m \quad f(x) = ab^{-m} = \frac{a}{b^m}$

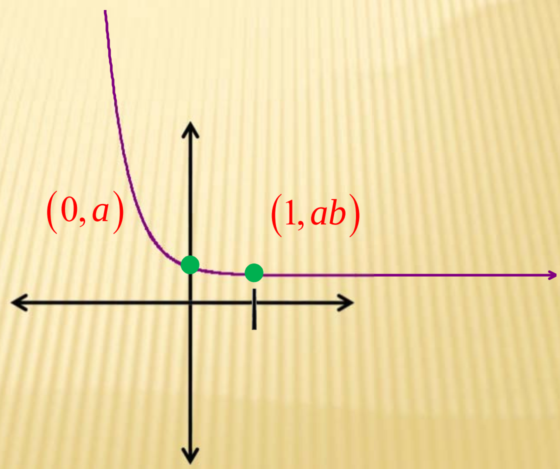
$x = -\frac{m}{n} \quad f(x) = ab^{-\frac{m}{n}} = \frac{a}{(\sqrt[n]{b})^m} = \frac{a}{(\sqrt[n]{b^m})}$

III. GRAPH OF $f(x) = ab^x$

Def: if $a > 0$ and $b > 1$ the $f(x)$ is increasing and is an exponential growth function with b as the growth factor.



Def: if $a > 0$ and $b < 1$ the $f(x)$ is decreasing and is an exponential decay function with b as the decay factor.



IV. RECURSIVE FUNCTIONS

Def: Any function which defines each term using the previous term. Exponential functions are recursive.

if $f(x) = ab^x$

$$f(x+1) = b \cdot f(x)$$

$$f(x+2) = b \cdot f(x+1)$$

$$f(x+3) = b \cdot f(x+2)$$

V. GRAPHICAL TRANSFORMATIONS

Describe how to transform $f(x) = 2^x$ into each of the following functions.

1.) $g(x) = 2^{x+1}$

-Trans. 1 unit left

2.) $g(x) = -3 \cdot 2^{2-x}$

- Refl. over the x
- Refl. over the y
- Vert. stretch by a factor of 3

- Trans. right 2

VI. THE NATURAL BASE *E*

Def:

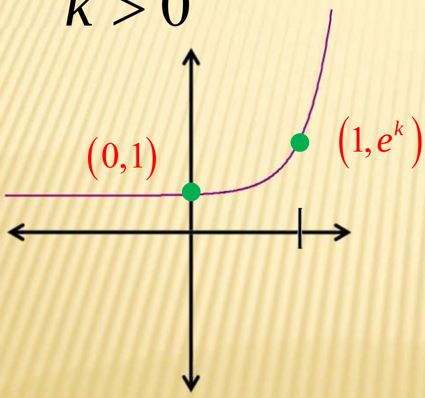
$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x \approx 2.71826...$$

$f(x) = e^x$ and $g(x) = \ln\ x$ are inverse functions

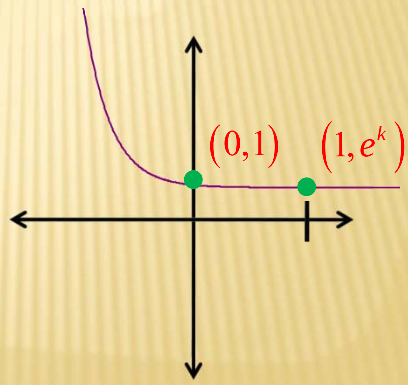
VII. GRAPHS OF $f(x) = e^x$

$f(x) = e^{kx}$

$k > 0$



$k < 0$



VIII. LOGISTIC FUNCTIONS

Def: Let *a*, *b*, *c*, and *k* be positive constants with *b*<1. A LOGISTIC GROWTH FUNCTION in *x* is a function that can be written in the form

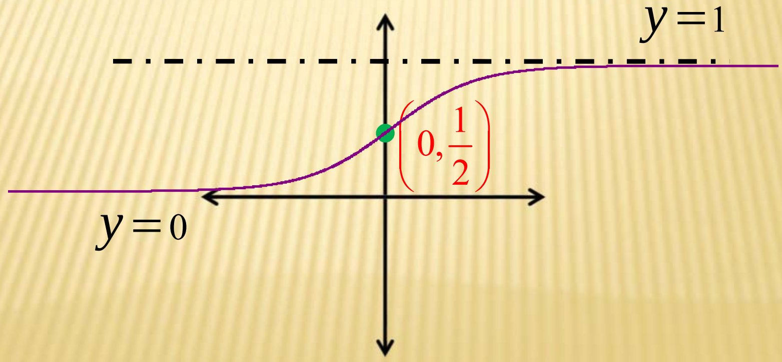
$$f(x) = \frac{c}{1 + a \cdot b^x} \text{ or } f(x) = \frac{c}{1 + a \cdot e^{-kx}}$$

where the constant *c* is the limit to the growth.

If *b*>1 or *k*<0, it is a logistic decay function.

IX. GRAPHING LOGISTIC FUNCTIONS

$$f(x) = \frac{1}{1+e^{-x}}$$

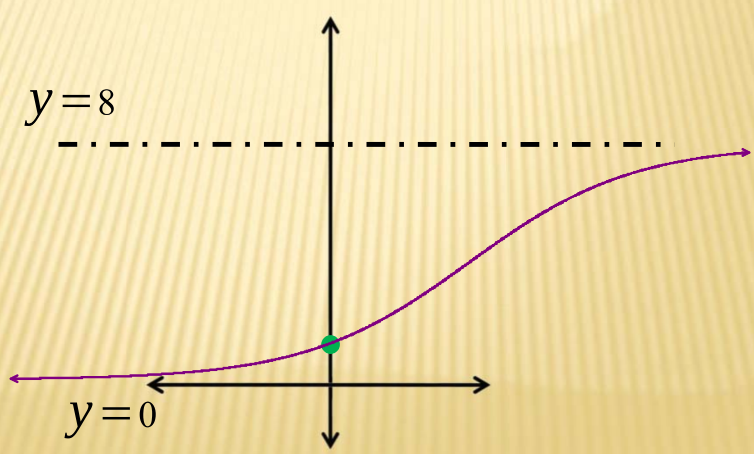


Graph

H.A.: $y=0; y=8$

$$f(x) = \frac{8}{1+3\cdot 0.7^{x}}$$

y-int: $(0,2)$



X. POPULATION MODELS

Exponential model: $$P(t) = P_0 b^t$$

Logistic Model - $$P(t) = \frac{c}{1 + P_0 \cdot b^t}$$

Given the population of San Jose in 1990 was 782,248 and in 2000 it was 894,943, determine an exponential model for the population of San Jose and predict San Jose's population in 2010.

1.) Find the growth factor.

$$b = \sqrt[10]{\frac{894,743}{782,248}} = 1.0135$$

2.) Determine the model.

$$P(t) = 782,248\left(1.0135^{t}\right)$$

3.) Substitute 20 in the model to determine the population in 2010.

$$P(20) = 782,248\left(1.0135^{20}\right) = 1,022,867$$

Example: Suppose that a long-lasting epidemic spreads through a town with a population of 100,000 according to the logistic function below:

$$N(t) = \frac{100,000}{1 + 5000e^{-t}}$$

Where *N(t)* is the number of infected individuals, and *t* is the number of days after the initial infection.

1. How much of the population will be infected after 5 days?

$$N(5) \approx 2,883\, people$$

2. After how many days will half of the townspeople be infected? (solve graphically)

$$8.5 days$$