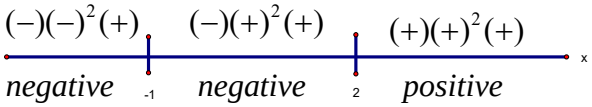
2-8: Solving Inequalities in One Variable

A. Example 1: Determine the values for which the function below is positive, negative, and zero

$$f(x) = (x-2)(x+1)^2(x^2+4)$$

The real zeros are:

$x = 2, -1$



$f(x) < 0$ when $x \in (-\infty, -1) \cup (-1, 2)$

$f(x) > 0$ when $x \in (2, \infty)$

$f(x) = 0$ when $x = -1, 2$

B. Example 2: Solve the following inequality analytically.

$$x^4 - 6x^2 - 8x - 3 > 0$$

Possible zeros are:

$\pm 1, \pm 3$

We find that -1 and 3 are zeros, so:

$x^4 - 6x^2 - 8x - 3 > 0$

$(x+1)(x^3 - x^2 - 5x - 3) > 0$

$(x+1)(x-3)(x^2 + 2x + 1) > 0$

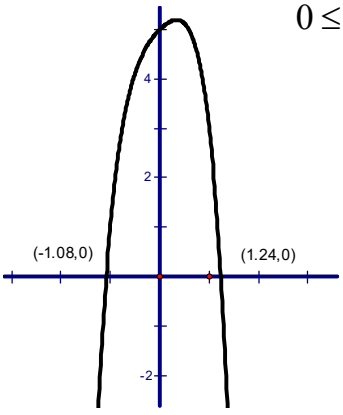
$$(-\infty, -1) \cup (3, \infty)$$

C. Example 3: Solve the inequality below graphically

$$2x^4 + x^2 \leq +x + 5$$

$$0 \leq -2x^4 - x^2 + x + 5$$

$$\boxed{x \in [-1.082, 1.24]}$$



D. Example 4: Compare the functions below to zero using each of the four inequality symbols.

$$f(x) = -\left(x^2+4\right)\left(x^2+1\right)$$

$f(x) > 0 \quad x \in \varnothing$

$f(x) \geq 0 \quad x \in \varnothing$

$f(x) < 0 \quad x \in (-\infty, \infty)$

$f(x) \leq 0 \quad x \in (-\infty, \infty)$

$$g(x) = \left(x^2+1\right)(x-2)^2$$

$g(x) > 0 \quad x \in (-\infty, 2) \cup (2, \infty)$

$g(x) \geq 0 \quad x \in (-\infty, \infty)$

$g(x) < 0 \quad x \in \varnothing$

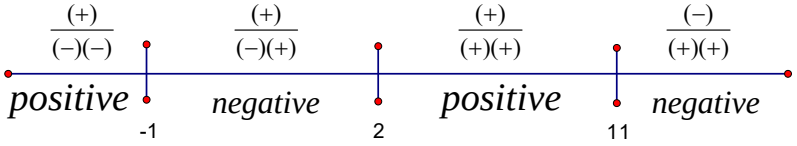
$g(x) \leq 0 \quad x \in \{2\}$

E. Example 5: Solve the inequality below by combining fractions and making a sign chart

$$\frac{3}{x-2} - \frac{4}{x+1} \geq 0$$

$$\frac{3(x+1)-4(x-2)}{(x-2)(x+1)} \geq 0$$

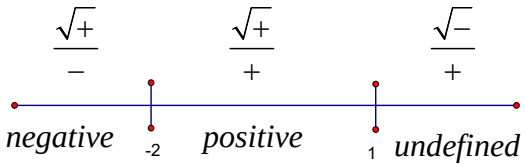
$$\frac{11-x}{(x-2)(x+1)} \geq 0$$



$$\boxed{x \in (-\infty, -1) \cup (2, 11]}$$

F. Example 6: Solve the inequality below:

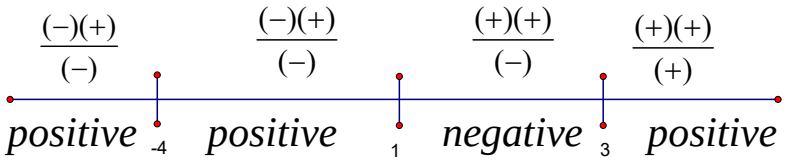
$$\frac{\sqrt{1-x}}{x+2} > 0$$



$$(-2,1)$$

G. Example 7: Solve the inequality below:

$$\frac{(x-1)|x+4|}{x-3} \le 0$$



$$x \in [-4] \cup [1,3)$$