



2-7: Solving Equations in one Variable

I. Solving Rational Equations

- A. Equations involving rational expressions or fractions are *rational equations*.

$$\frac{f(x)}{g(x)} = 0$$

- B. When we multiply or divide an equation by an expression containing variables, we may have solutions that are *not* solutions of the original equation. These are called *extraneous solutions*.

C. Solve each of the following:

Example 1 $\frac{x-4}{4} + \frac{x}{3} = 6$

$$12\left(\frac{x-4}{4} + \frac{x}{3}\right) = 12(6)$$

$$3(x-4) + 4(x) = 72$$

$$3x - 12 + 4x = 72$$

$$\boxed{x = 12}$$

Example 2 $\frac{3}{2x} - \frac{2x}{x+1} = -2$ note: $x \neq 0, -1$

$$(2x)(x+1)\left(\frac{3}{2x} - \frac{2x}{x+1}\right) = -2(2x)(x+1)$$

$$3(x+1) - 2x(2x) = -4x^2 - 4x$$

$$3x + 3 - 4x^2 = -4x^2 - 4x$$

$$3x + 3 = -4x$$

$$3 = -7x$$

$$\boxed{-\frac{3}{7} = x}$$

Example 3 $x - \frac{2}{x-3} = \frac{x-1}{3-x}$ note: $x \neq 3$

$$x - \frac{2}{x-3} = \frac{x-1}{-(x-3)}$$

$$(x-3)\left(x - \frac{2}{x-3}\right) = (x-3)\left(\frac{x-1}{-(x-3)}\right)$$

$$(x-3)x - 2 = -(x-1)$$

$$x^2 - 3x - 2 = 1 - x$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\cancel{x=3}, \boxed{x=-1}$$

Example 4 $\frac{2x}{x-1} + \frac{x-5}{x^2-1} = 1$ note: $x \neq \pm 1$

$$\frac{2x}{x-1} + \frac{x-5}{(x-1)(x+1)} = 1$$

$$(x-1)(x+1)\left(\frac{2x}{x-1} + \frac{x-5}{(x-1)(x+1)}\right) = (x-1)(x+1)1$$

$$2x(x+1) + x - 5 = x^2 - 1$$

$$2x^2 + 2x + x - 5 = x^2 - 1$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$\boxed{x=-4}, \cancel{x=1}$$