Notes 2.6 – Rational Functions

I. Definition:

- Let *f* and *g* be polynomial functions with $g(x) \neq 0$, then

$$y(x) = \frac{f(x)}{g(x)}$$

is a **rational function**.

II. Vertical and Horizontal Asymptotes

- Def: The line $x = a$ is a vertical asymptote of the graph of the function f iff

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty$$

- Def: The line $y = b$ is a horizontal asymptote of the graph of the function f iff

$$\lim_{x \to \pm\infty} f(x) = b$$

III. Basic Rational Function

$$f(x) = \frac{1}{x}$$

10 Characteristics?

IV. Transformations

- Example – Describe how $g(x)$ can be obtained by transforming $f(x)=\frac{1}{x}$.

A.) $g(x)=\frac{3}{x-4}$

B.) $g(x)=\frac{2x-5}{x+7}$

-Vertically Stretch by 3

- Translate right 4

$$g(x)=\frac{2x-5}{x+7} = 2-\frac{19}{x+7}$$

- Reflect over the x
- Vert. Stretch by 19
- Trans. left 7
- Trans. up 2

V. Limits and Asymptotes

- Ex. Find the vertical and horizontal asymptotes for each of the following and describe the behavior at each vertical asymptote.

A.) $f(x)=\frac{2x^2-1}{x^2+3}$ B.) $f(x)=\frac{x-3}{x+3}$

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- V.A. – None
- H.A. $y = 2$ Why?

$$\lim_{x\to\pm\infty} f(x)=\frac{\frac{2x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{3}{x^2}}=$$

$$\lim_{x\to\pm\infty}\frac{2-\frac{1}{x^2}}{1+\frac{3}{x^2}}=\frac{2-\frac{1}{\pm\infty}}{1+\frac{3}{\pm\infty}}=\frac{2-0}{1+0}=2$$

B.) $f(x)=\frac{x-3}{x+3}$

- V.A.: $x = -3$
- H.A.: $y = 1$

$$\lim_{x\to -3^{+}} = \frac{-3^{+}-3}{\left(-3^{+}\right)+3} = \frac{-6}{0^{+}} = -\infty$$

$$\lim_{x\to -3^{-}} = \frac{-3^{-}-3}{\left(-3^{-}\right)+3} = \frac{-6}{0^{-}} = \infty$$

$$\lim_{x\to \pm\infty} = \frac{x-3}{x+3} = \frac{\frac{x}{x}-\frac{3}{x}}{\frac{x}{x}+\frac{3}{x}} =$$

$$\lim_{x\to \pm\infty} = \frac{1-\frac{3}{x}}{1+\frac{3}{x}} = \frac{1-0}{1+0} = 1$$

VI. Slant Asymptotes

- Def – The **end behavior** asymptote of a rational function when the highest power is in the numerator.

Ex – Find the slant asymptote of

$$f(x)=\frac{x^2+2x-3}{x+2}$$

$$x+2\overline{)x^2+2x-3} = x-\frac{3}{x+2}$$

- Therefore, the line $y = x$ is the **slant asymptote** of the graph of $f(x)$.

This means the graph of $f(x)$ will act like the line $y = x$ as x approaches $\pm\infty$

Ex – Using intercepts and asymptotes graph

$$f(x)=\frac{x^2+2x-3}{x+2}$$

x-intercepts : (-3, 0) and (1, 0)

y-intercept: (0, -3/2)

V.A.: $x = -2$

H.A.: None

S.A.: $y = x$

- How do we graph it?

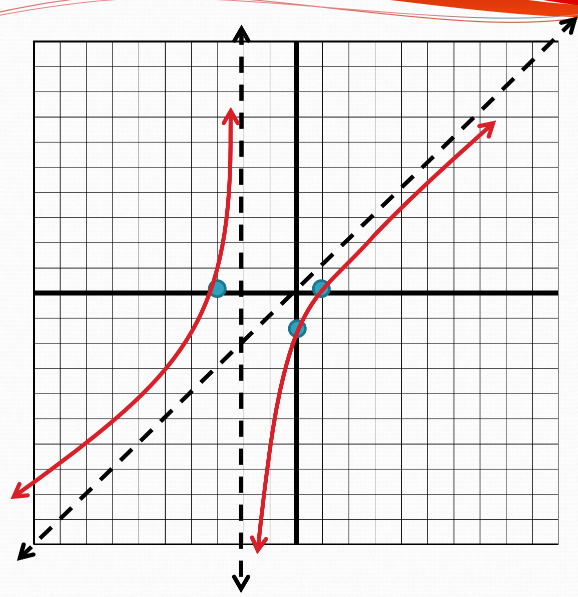
1.) Plot the intercepts on the plane.

2.) Draw dashed lines for all asymptotes.

3.) Determine the behaviors at each V.A.

$$\lim_{x\to -2^+} = \frac{(-2^+ + 3)(-2^+ - 1)}{\left(-2^+\right) + 2} = \frac{-3}{0^+} = -\infty$$

$$\lim_{x\to -2^-} = \frac{(-2^- + 3)(-2^- - 1)}{\left(-2^-\right) + 2} = \frac{-3}{0^-} = \infty$$



$$f(x)=\frac{x^4-2x+1}{x-2}$$

- V.A. – $x=2$
- H. A. – None
- S. A. – $y=x^3+2x^2+4x+6$

- Ex – Find all the asymptotes of

$$f(x)=\frac{x^4-2x+1}{x-2}$$

Ex-

- Analyze and graph

$$f(x) = \frac{2x^2 - 2}{x^2 - 4}$$

Ex-

- Analyze and graph

$$f(x) = \frac{x^3 - 3x^2 + 3x + 1}{x - 1}$$