Notes 2.3 – Polynomials of Higher Degree

I. Polynomial Functions:

A.) Standard Form: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

B.) Cubic Functions – Poly. of degree 3

C.) Quartic Functions – Poly. of degree 4

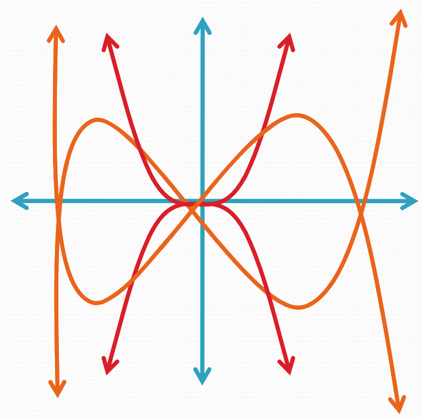
D.) All polynomial functions are smooth, continuous curves. No “jumps”, “corners”, or “cusps”.

II. Cubic Functions

A.) General Form: $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

B.) $a_3 > 0$:

C.) $a_3 < 0$:

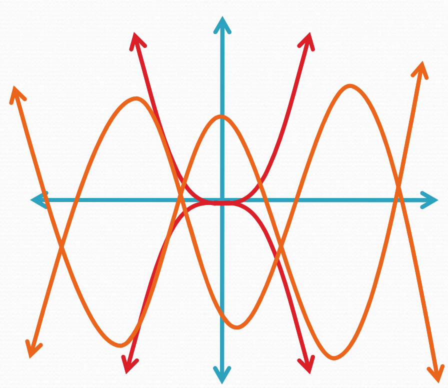


III. Quartic Functions

A.) General Form: $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

B.) $a_4 > 0$:

C.) $a_4 < 0$:



IV. Comparing Graphs of Polys and Their Leading Terms

A.) Using your TI-83+, graph the following on the same axis: $y = x^3 - 4x^2 - 5x - 3$ $y = x^3$

Zoom out until both graphs look alike.

All Polynomial functions will model the same behavior as the function's leading term.

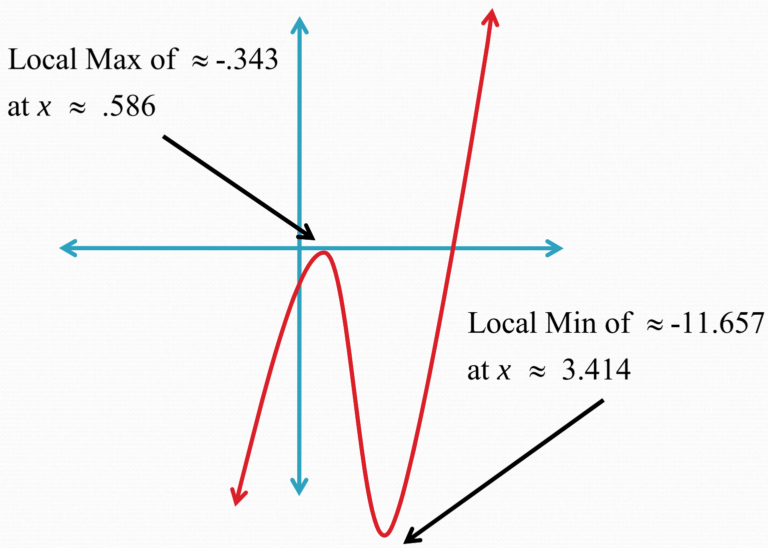
V. Local Extrema

A.) Theorem: A poly. Fn. of degree *n* has at most *n*-1 local extrema and at most *n* zeros.

B.) Ex – Find any local extrema on the following graph:

$$f(x) = x^3 - 6x^2 + 6x - 2$$

Solution:



VI. End Behavior

A.) Related to the E.B. of the leading term.

B.) Cubic - $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

$a_3 > 0$: $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = -\infty$

$a_3 < 0$: $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$

C.) Quartic - $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

$a_4 > 0$: $\lim_{x\to\pm\infty} f(x) = \infty$

$a_4 < 0$: $\lim_{x\to\pm\infty} f(x) = -\infty$

VII. Zeros of Polynomials

A.) Zero of a function – The *x* value when $y = 0$

B.) Set $f(x) = 0$ and solve.

C.) Multiplicity of a Zero of a Poly. Fn. –

If f is a poly. fn. and $(x-c)^m$ is a factor of f but $(x-c)^{m+1}$ is not, then c is a zero of multiplicity m of f.

1.) Even Multiplicity – Graph does not cross the *x*-axis.

2.) Odd Multiplicity – Graph does cross the *x*-axis.

1.) Ex- Consider the function $f(x) = (x+2)^4 (x-1)^3$

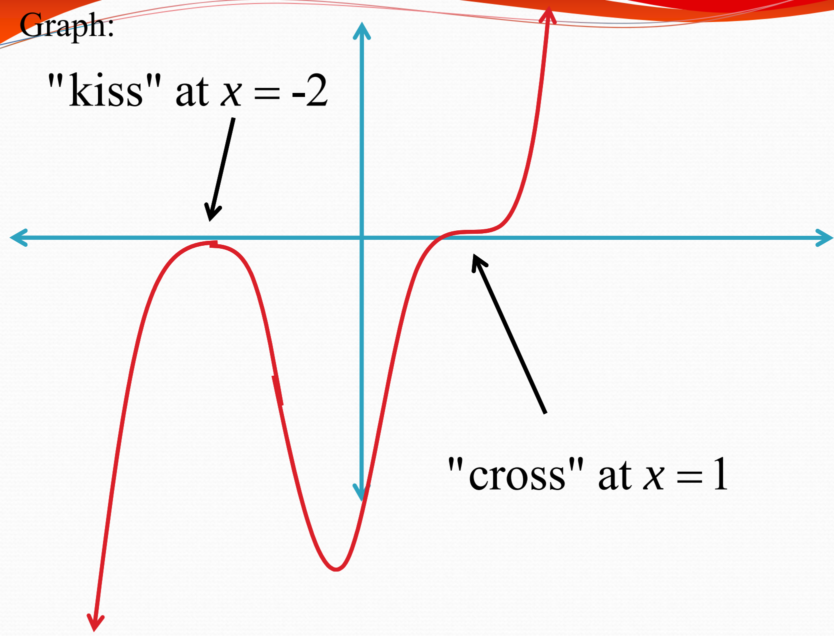
The zeros are -2 and 1.

-2 has a multiplicity of 4, and 1 has a multiplicity of 3.

Therefore, the graph will touch, but not cross, the *x*-axis at $x = -2$, and cross the *x*-axis at $x = 1$.

Since the leading term will be x^7, the graph will start in the 3rd quadrant, and end in the first.

Graph:



VIII. Intermediate Value Thm.

If a and b are real numbers and f is continuous on $[a, b]$, then f takes on every value between $f(a)$ and $f(b)$. In other words, if y is between $f(a)$ and $f(b)$, then $y = f(c)$ for some number c in $[a, b]$.

Cor: If $f(a)$ and $f(b)$ have opposite signs, then $f(c) = 0$ for some number c in $[a, b]$.

Prove $f(x) = x^3 - 2x + 1$ has at least 1 real zero.

Polynomial? Yes!

Pick two numbers that may give you opposite signs.

$$f(2) = 2^3 - 2(2) + 1 \ = 5$$

$$f(-2) = -2^3 - 2(-2) + 1 \ = -3$$

$\therefore \ f(x) = 0$ at least once between -2 and 2.

IX. Modeling

- We want a "good fit" – try to find a balance between good fit and simplicity. Generally, try to use an accurate model of the lowest degree.