2-2: Power Functions With Modeling

Power Functions:

- Any function that can be written in the form:

$$f\left(x\right)=kx^{a}, \text{ where } k \text{ and } a \text{ are constants}$$

  - *a* is the **power**
  - *k* is the **constant of variation**(proportion)

- We say $f(x)$ **varies** as the a^{th} power of *x*.

Examples of Power Functions				
Name	Formula	Power	Constant of Variation	Read as
Circumference	$C = 2\pi r$	1	2π	*C* varies directly as *r*.
Volume of a Sphere	$A = \frac{4}{3}\pi r^3$	3	$\frac{4}{3}\pi$	V varies directly as the cube of *r*.
Force of Gravity	$F = \frac{k}{d^2}$	−2	*k*	*F* varies inversely as the square of *d*.
Boyle's Law	$V = \frac{k}{P}$	−1	*k*	*V* varies inversely as *P*.

Power Functions:

- Direct Variation
  - As the **independent** variable increases, the **dependent** variable **increases**.

Examples: $C = 2\pi r$ $A = \pi r^2$ $V = s^3$

- Inverse Variation
  - As the **independent** variable increases, the **dependent** variable **increases**.

Examples: $F = \frac{k}{d^2}$ $V = \frac{k}{P}$

Note: The power function formulas with positive powers are statements of *direct variation* and power function formulas with negative powers are statements of *inverse variation*. Unless the word *inversely* is included, it is assumed that the *variation* is direct.

B. Basic power functions:

You have already been introduced to the following power functions: $x, x^2, x^3, x^{-1} = \frac{1}{x}, x^{\frac{1}{2}} = \sqrt{x}$

C. Examples

1. The period of time *T* for the full swing of a pendulum varies as the square root of the pendulum's length *l*. Express this relationship as a power function.

$$T = k\sqrt{l} = kl^{\frac{1}{2}}$$

Example 2: Analyze the following functions

a.	$f(x) = \sqrt[3]{x}$	b.	$g(x) = 1/x^2$
	Power: $1/3$		Power: -2
	C.o.v.: 1		C.o.v.: 1
	D: $(-\infty, \infty)$		D: $(-\infty, 0) \cup (0, \infty)$
	R: $(-\infty, \infty)$		R: $(0, \infty)$
	C: continuous for all x		C: infinite discontinuity at $x = 0$
	I/D: increasing for all x		I/D: inc: $(-\infty, 0)$, dec: $(0, \infty)$
	S: odd		S: even
	B: not bounded above or below		B: bounded below
	E: no local extrema		E: no local extrema
	A: no asymptotes		A: v.a.: $x = 0$, h.a.: $y = 0$
	E.B: $\lim_{x\to-\infty} \sqrt[3]{x} = -\infty, \lim_{x\to\infty} \sqrt[3]{x} = \infty$		E.B: $\lim_{x\to-\infty} 1/x^2 = 0, \lim_{x\to\infty} 1/x^2 = 0$

Graphs of Power Functions

- Monomial Function
  - Any function that can be written as:

$$f(x) = k \text{ or } f(x) = kx^n$$

where *k* is a constant and *n* is a positive integer

- Comparing the graphs of monomial functions
  - Compare the monomials where *n = 1,2,3,4,5, and 6*, in the standard window, as well as the window x: [0,1] and y: [0,1]. Make any observations.

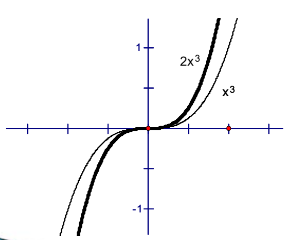
Graphing Monomial Functions

Describe how to obtain the graph of the given function from its power parent function. Then sketch the graph of the function.

a. $f(x) = 2x^3$

Parent function: $f(x) = x^3$

vertically stretch by a factor of 2

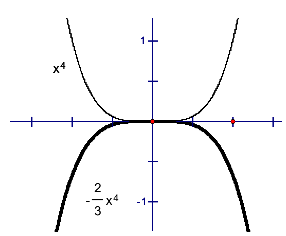


b. $f(x) = -\frac{2}{3}x^4$

Parent function: $f(x) = x^4$

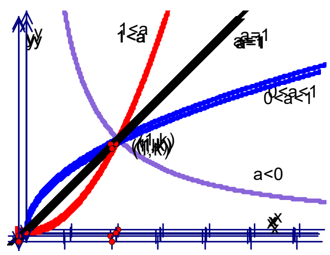
vertically shrink by a factor of 2/3

reflect over x-axis

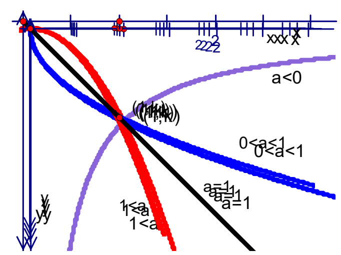


The graphs below represent the four general shapes that are possible for power functions of the form $f(x) = kx^a$ for $x \geq 0$.

a. $k > 0$



b. $k < 0$



Graphing Power Functions $f(x) = kx^a$

Describe the following graphs without using your calculator. Graph to check your accuracy.

Function	Description
a. $f(x) = 2x^{-3}$	Because k = 2, the graph passes through $(1,2)$ Because a < 0, the graph is asymptotic to both axes The function is odd
b. $f(x) = -0.4x^{1.5}$	Because k = -0.4, the graph passes through $(0,\ 0)$ and $(1,-0.4)$ The function is undefined for x < 0.
c. $f(x) = -x^{.4}$	since k = -1 and is negative and 0 < a < 1, the graph contians (0,0) and (1,-1). In the fourth quadrant, it is decreasing The function is even.

Modeling with Power Functions

Kepler's third law of planetary motion states that the period of orbit (T) is related to the distance a from the sun according to the equation:

$$T(a) \approx ka^{\frac{3}{2}}$$

Average distances and orbit periods for the six innermost planets		
Planet	Average distance from the Sun (Gm)	Period of Orbit (days)
Mercury	57.9	88
Venus	108.2	225
Earth	149.6	365.2
Mars	227.9	687
Jupiter	778.3	4332
Saturn	1427	10760

Modeling with Power Functions

A. Find a power model for the orbital period:

$$T(a) \approx 0.20a^{1.5}$$

B. Graph the data as well as the power model on your calculator to see how they fit.

C. Predict the orbital period for Neptune, which is 4497 Gm from the Sun on Average

60313.47 days

D. Compare this to the actual answer, which is about 60313

Homework: p.182 #3-48 (multiples of 3), 49-63