

## Notes 2.1 – Linear and Quadratic Functions

### I. Polynomial Functions:

A.) Polynomial Function: Let  $n$  be a nonnegative integer and let  $a_0, a_1, a_2, \dots, a_{n-1}, a_n$

be real numbers with  $a_n \neq 0$ .

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

is a **polynomial function** of **degree  $n$**  with a **leading coefficient  $a_n$** .

B.) Examples:

Polynomials:

$$f(x) = 3x^3 + 2x^2 - x + 1 \quad f(x) = 15x - 2x^4$$

Non-polynomials:

$$f(x) = 6x^{-3} \quad f(x) = \sqrt{6 - x^4}$$

C.) Poly. Fns. Of Low and No Degree –

Name	Form	Degree
Zero	$f(x)=0$	Undefined
Constant	$f(x)=a, a \neq 0$	0
Linear	$f(x)=ax+b, a \neq 0$	1
Quadratic	$f(x)=ax^2+bx+c, a \neq 0$	2

## II. Linear Equations $f(x) = ax + b$ , $a \neq 0$

A.) Terms:

- 1.)  $a$  is **slope** of the line.
- 2.)  $a$  is also the **constant rate of change** of the equation.
- 3.)  $b$  is the **y-intercept** of the graph of the equation.

B.) Finding the equation of a line:

Ex- Find the equation of the line passing through the points  $(-1, 2)$  and  $(5, 9)$  in point-slope form.

PT/SLOPE FORM:  $y - y_1 = m(x - x_1)$

$$m = \frac{9-2}{5-(-1)} = \frac{7}{6}$$

$$y - 9 = \frac{7}{6}(x - 5)$$

$$y - 2 = \frac{7}{6}(x + 1)$$

C.) Rate of Change:

Average Rate of Change in  $f$  from  $a$  to  $b$  is

$$\frac{f(b) - f(a)}{b - a}$$

D.) Ex. Find the average rate of change of  $f$  from  $x = 2$  to  $x = 4$  of  $f(x) = \frac{1}{2}x^2 + 3$

$$= \frac{f(4) - f(2)}{4 - 2} = \frac{11 - 5}{2} = 3$$

### III. Linear Modeling and Correlation

A.) Linear Correlation – How close the data is to the regression line.

1.) Positive Linear – Strong/Weak

2.) Negative Linear – Strong/Weak

3.) Little or No Correlation

B.) The Correlation Coefficient  $r$ :

Properties of  $r$

1.)  $-1 \leq r \leq 1$

2.) If  $r > 0$ , POSITIVE CORRELATION

3.) If  $r < 0$ , NEGATIVE CORRELATION

4.) If  $|r| = 1$ , PERFECT CORRELATION

5.) If  $|r| = 0$ , NO CORRELATION

## IV. Quadratic Functions

A.) Polynomial of degree 2.

B.) Forms:

1.) Standard Form:

$$f(x) = ax^2 + bx + c$$

2.) Vertex Form:

$$f(x) = a(x - h)^2 + k$$

Vertex:

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

$$(h, k)$$

Axis:

$$x = -\frac{b}{2a}$$

$$x = h$$

Max/Min:

$$y = f\left(-\frac{b}{2a}\right)$$

$$y = k$$

1.) Standard Form:      2.) Vertex Form:

$$f(x) = ax^2 + bx + c \quad f(x) = a(x-h)^2 + k$$

y-intercept:               $(0, c)$                $(0, f(0))$

x-intercept:               $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$                $\left(\pm \sqrt{\frac{-k}{a}} + h, 0\right)$

C.) Ex- Find the vertex, axis of symmetry, max/min, y-int., x-int., of the following and rewrite in vertex form.

$$f(x) = 6x - 3x^2 - 5$$

v = (1, -2)                              x - int. = DNE

a: x = 1                                      vertex form:

max of -2 at x = 1                       $f(x) = -3(x-1)^2 - 2$

y - int. = (0, -5)

D.) Ex- Complete the square to rewrite the following in vertex form.

$$f(x) = 2x^2 - 4x + 13$$

vertex form:

$$f(x) = 2(x-1)^2 + 11$$

## V. Free-Fall

A.) The position(height) of an object is given by the equations:

$$1.) s(t) = -16t^2 + v_0t + s_0 \quad 2.) s(t) = -4.9t^2 + v_0t + s_0$$

where  $t$  is in secs. and  $s$  is in ft.      where  $t$  is in secs. and  $s$  is in m.

B.) The velocity of an object is given by the equations:

$$1.) v(t) = -32t + v_0 \quad 2.) v(t) = -9.8t + v_0$$

where  $t$  is in secs. and  $v$  is in ft./sec      where  $t$  is in secs. and  $v$  is in m/sec.

C.) Speed =  $|v(t)|$

D.) Ex- Johnny throws a rock directly upward with an initial velocity of 48 ft./sec., from an initial height of 9 ft. Find the following:

- 1.) How long is the rock in the air?
- 2.) What is the maximum height the rock reaches?
- 3.) What is the velocity of the rock when it hits the ground?

1.) How long is the rock in the air?

$$s(t) = -16t^2 + 48t + 9 \quad 0 = -16t^2 + 48t + 9$$

$$t \approx 3.177 \text{ sec}$$

2.) What is the maximum height the rock reaches?

$$v \approx (1.5, 45) \quad \therefore 45 \text{ ft.}$$

3.) What is the velocity of the rock when it hits the ground?

$$v(t) = -32t + 48 \quad v(3.177) = -32t + 48 = -53.664 \frac{\text{ft}}{\text{sec}}$$