Notes 10.4 – Random Variables and Probability Models – Day 1

I. Expected Values

A.) Random Variable

1.) Def: That which assumes numerical values as a result of some random event.

2.) Notation: Random variables are denoted by capital letters.

Ex: Let *X* equal the result of rolling a six-sided die.

$$X = \{1, 2, 3, 4, 5, 6\}$$

II. Probability Model

A.) Def. – A PROBABILITY Model is a random variable *X* together with the function that assigns to each possible outcome a probability $P(x_i)$ such that $\sum_{i=1}^{n} P(x_i) = 1.$

Ex:

X	1	2	3	4	5	6
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

B.) Expected Value of a Random Variable:

Def: The mean of the probability model, or the sum of the values of *X* multiplied by their corresponding probabilities.

$$\mu = E(X) = \sum_{i=1}^{n} x_i \left(P(x_i)\right)$$

Example: Some fantasy games use a 12-sided die. Suppose one such die has a 1 on seven of the faces, a 5 on three faces, and a 10 on the remaining two faces. What is the expected value of a roll of this die?

$$\mu = E(X) = 1\left(\frac{7}{12}\right) + 5\left(\frac{3}{12}\right) + 10\left(\frac{2}{12}\right) = 3.5$$

Example: Why card games are illegal at North: Suppose your huckster friend is playing a card game during lunch. You pay $5 to pick a random card from a deck. If it's black, you lose, and he keeps the money. If it is a heart, you win your $5 back. If it is any diamond except the ace, you win $10. What should be the prize for drawing the ace of diamonds in order to make the game fair? ($E(X) = 0$)

$$E(X) = 0 = -5\left(\frac{26}{52}\right) + (-5+5)\left(\frac{13}{52}\right) + (-5+10)\left(\frac{12}{52}\right) + (-5+A)\left(\frac{1}{52}\right)$$

$$E(X) = 0 = -5\left(\frac{1}{2}\right) + 0\left(\frac{1}{4}\right) + 5\left(\frac{3}{13}\right) + (-5-A)\left(\frac{1}{52}\right)$$

$$A = \$75$$

III. Binomial Distributions

A.) Def: A probability model for the random variable X where X = the number of successes in n trials given the following requirements:

1.) Each outcome has only two possible outcomes, success (p) or failure (q).

2.) The probability of success on each trial remains constant

3.) The trials are independent

It is denoted $P(X = k) = \binom{n}{k} \cdot p^k \cdot q^{n-k}$

B.) Ex. – You roll 6 fair dice at one time. Find the probability you rolled exactly 4 twos.

$$P(4-2s)=\binom{6}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2=15\left(\frac{25}{46,656}\right)=\frac{125}{15,552}\approx 0.00804$$

C.) Ex. - Find the probability of rolling at least 1 two.

$$P(1+2s)=1-\binom{6}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6 \qquad =1-\left(\frac{15,625}{46,656}\right)\approx 0.665$$

D.) Ex. - Find the probability of rolling at least 2, but at most 5 two's. $P(2\le x\le 5)=P(2)+P(3)+P(4)+P(5)$

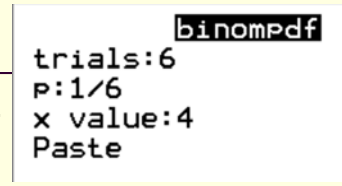
$$P(2\le x\le 5)=\frac{12,280}{46,656}=\frac{1535}{5832}\approx 0.2632$$

Number of 2's	Probability
6	${}_6C_6\cdot\left(\frac{1}{6}\right)^6\cdot\left(\frac{5}{6}\right)^0=\frac{1}{46656}$
5	${}_6C_5\cdot\left(\frac{1}{6}\right)^5\cdot\left(\frac{5}{6}\right)^1=\frac{5}{7776}$
4	${}_6C_4\cdot\left(\frac{1}{6}\right)^4\cdot\left(\frac{5}{6}\right)^2=\frac{125}{15,552}$
3	${}_6C_3\cdot\left(\frac{1}{6}\right)^3\cdot\left(\frac{5}{6}\right)^3=\frac{625}{11,664}$
2	${}_6C_2\cdot\left(\frac{1}{6}\right)^2\cdot\left(\frac{5}{6}\right)^4=\frac{3125}{15,552}$
1	${}_6C_1\cdot\left(\frac{1}{6}\right)^1\cdot\left(\frac{5}{6}\right)^5=\frac{3125}{7776}$
0	${}_6C_0\cdot\left(\frac{1}{6}\right)^0\cdot\left(\frac{5}{6}\right)^6=\frac{15,625}{46,656}$

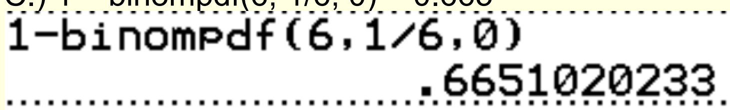
E.) Now, Examples B.), C.), and D.) on the calculator…

B.) DISTR – 0:binompdf

binompdf(6, 1/6, 4) = 0.00804



C.) 1 – binompdf(6, 1/6, 0) = 0.665



D.) DISTR – 1:binomcdf

binomcdf(6, 1/6, 5) – binomcdf(6, 1/6, 1) =

0.2632