Notes 10.1 – Probability

I. Sample Spaces and Probability Functions
   A. Vocabulary:
      1. The Sample Space is all possible events
      2. An event is a subset of the sample space
      3. Probability: If *E* is an event in a finite, nonempty sample space *S*, then the *probability* of the event *E* is:

$$P(E) = \frac{\text{The number of equally likely outcomes in } E}{\text{The number of equally likely outcomes in } S}$$

or

$$P(E) = \frac{N(E)}{N(S)}$$

B. Probability Distribution: each event is assigned a unique probability.

The distribution below shows the distribution for the number of heads (or tails) flipped when flipping a coin four times:

Outcome	Probability
0	1/16
1	4/16 = 1/4
2	6/16 = 3/8
3	4/16 = 1/4
4	1/16

C. These values can be found using the nCr function. We must find both N(E) and N(S).

1. In order to find N(2) for example, we must choose 2 from 4, because we are choosing two of the coin flips to be heads (first and second, first and third, first and fourth, etc.). The six possibilities are as follows:

HHTT, HTHT, HTTH, THHT, THTH, TTHH

2. The denominator contains all possible subset. Therefore, we must use 2^n.

$$P(2) = \frac{N(E)}{N(S)} = \frac{{}_4C_2}{2^4} = \frac{6}{16} = \boxed{\frac{3}{8}}$$

D. Example 1: Make the probability distribution for flipping a coin 5 times.

$$N(S) = 2^5 = 32$$

Outcome	Number of ways	Probability
0		
1		
2		
3		
4		
5		

D. Example 2: What is the probability of flipping fewer than 4 heads when flipping a coin 5 times.

$$P(n<4) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} = \frac{26}{32} = \boxed{\frac{13}{16}}$$

Or

$$P(n<4) = 1 - P(n \geq 4) = 1 - \left(\frac{5}{32} + \frac{1}{32}\right) = \frac{26}{32} = \boxed{\frac{13}{16}}$$

E. Definition: Probability Function

A probability function is a function *P* that assigns a real number to each outcome in a sample space *S* subject to the following conditions:

1. $0 \le P(E) \le 1$ for every event P
2. $\sum P(E) = 1$
3. $P(\varnothing) = 0$

F. Probability of an Event (Outcomes not equally likely)

Let *S* be a finite, nonempty sample space in which every outcome has a probability assigned to it by a probability function *P*. If *E* is any event in *S*, the *probability of* the event *E* is the sum of the probabilities of all of the outcomes contained in *E*.

B. Multiplication principle of probability:

Suppose an event *A* has probability p_1 and an event *B* has probability p_2 under the assumption that *A* occurs. Then the probability that both *A* and *B* occurs is $p_1 p_2$.

C. Example 4: Two balls are drawn at random from a bag containing 6 red and 10 green balls. What is the probability of drawing 2 red balls?

There are four possible events; rr, rg, gr, and gg. We want P(rr), which can be found as follows:

$$P(rr) = \frac{6}{16} \cdot \frac{5}{15} = \boxed{\frac{1}{8}}$$

$$P(E) = \frac{N(E)}{N(S)} = \frac{{}_6C_2 \cdot {}_{10}C_0}{{}_{16}C_2} = \frac{15 \cdot 1}{120} = \boxed{\frac{1}{8}}$$

Types of Events

A.) DEPENDENT EVENT - The probability of the second event occurring depends on the probability of the first event occurring. (Without Replacement)

The last example was a dependent event.

B.) INDEPENDENT EVENT - The probability of the first event occurring has no effect on the probability of the second event occurring. (With Replacement)

Ex. – Rolling a die and then flipping a coin

Ex 2- SPSE there are 2 jars that each contain 4 cookies. Jar A contains 2 chocolate chip and 2 peanut butter. Jar B contains 3 chocolate chip and 1 peanut butter. What is the probability of choosing a chocolate chip cookie?

Easy, right? Simply put, since there are four cookies in each jar and we have an equal choice of choosing either jar, we have

$$\frac{5}{8} = .625$$

But what if Jar B had only 1 cookie and it was a chocolate chip? (And no, it is not 0.6.)

Lets look at a tree diagram…

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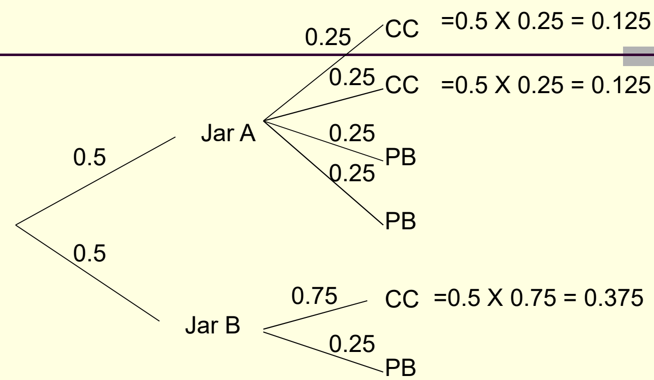


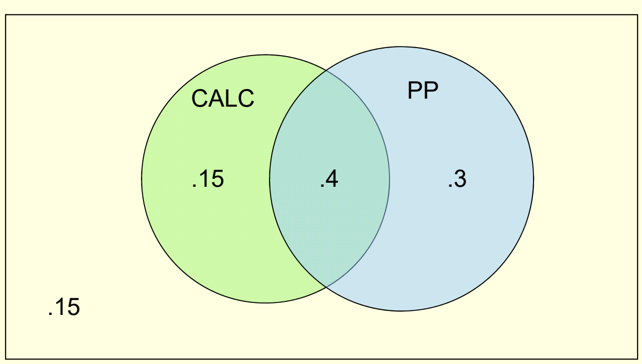
$$= P(\text{A and CC}) + P(\text{A and CC}) + P(\text{B and CC})$$

$$0.125 + 0.125 + 0.375 = 0.625$$

Venn Diagrams

A graphical representation of events within a sample space. Be sure that the sum of the percents adds to 100%.

Ex- Of the 1000 math teachers, 70% have a pocket protector, 55% have a calculator and 400 have both. Draw a Venn diagram representing this situation.



Conditional Probability

A.) *P*(*B*|*A*)→ the probability of event B occurring given that event A has already occurred.

$$P(A\&B)=P(A)\bigcap P(B\mid A)$$

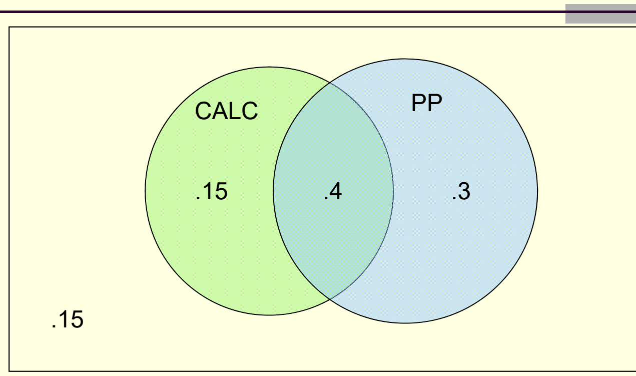
Therefore…

$$P(B\mid A)=\frac{P(A\&B)}{P(A)}$$

B.) Remember our cookie problem? What is the probability that, given we selected a chocolate chip cookie, it came from Jar B?

$$P(B\mid CC)=\frac{P(B\&CC)}{P(CC)}=\frac{.5(1)}{.75}=\frac{2}{3}$$

From our Venn Diagram, what is the probability that the math teacher has a calculator given he is wearing a pocket protector.



$$P(C\mid P)=\frac{P(C\&P)}{P(P)}=\frac{.4}{.7}=\frac{4}{7}$$

Independence of an Event

If events A and B are independent, then

$$P(A) = P(A | B) \quad \text{and} \quad P(B) = P(B | A)$$

Ex- Is having a calculator independent of having a pocket protector??

$$\text{Does } P(C) = P(C | P)?$$

$$P(C) = .55 \qquad P(C | P) = \frac{4}{7}$$

No, these events are not independent.