1-5: Parametric Equations and Inverses

Honors Precalculus
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Expressing Functions of Time

Suppose a balloon is inflated at a rate of 35 cu. mm./sec.:

1.) Find the volume of the balloon after *t* seconds.

$$V = 35t$$

2.) When the volume is V, what is the radius?

$$V = \frac{4}{3}\pi r^3 \qquad r = \sqrt[3]{\frac{3V}{4\pi}}$$

3.) Write an equation for the radius of the balloon as a function of time.

$$V = 35t \qquad r = \sqrt[3]{\frac{3V}{4\pi}} \qquad r = \sqrt[3]{\frac{105t}{4\pi}}$$

Implicitly Defined Relations

- Relation
  - A set of ordered pairs
- Implicitly Defined Relations
  - A relation whose equations defines two or more functions

$$x^2 + y^2 = 9$$

$$y^2 = 9 - x^2$$

$$y = \pm\sqrt{9 - x^2}$$

$$y = \sqrt{9 - x^2} \qquad y = -\sqrt{9 - x^2}$$

Parametric Relations

- Any relation where both elements in an ordered pair are defined in terms of time *t*.
- *t* is called the **parameter**.

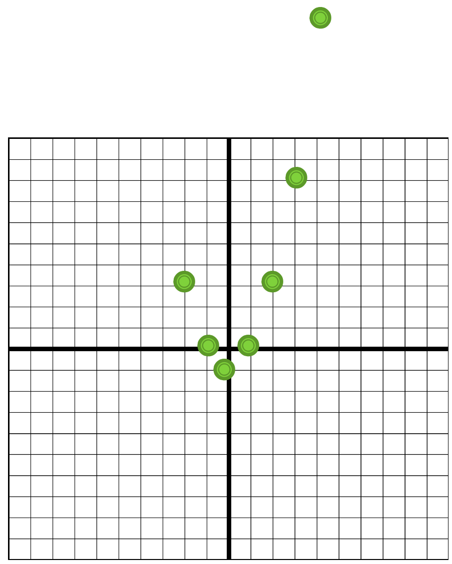
Consider the following parametric relation:

$$x = t + 1$$

$$y = t^2 + 2t$$

Find (x,y) for t = {-3,-2,-1,0,1,2,3}

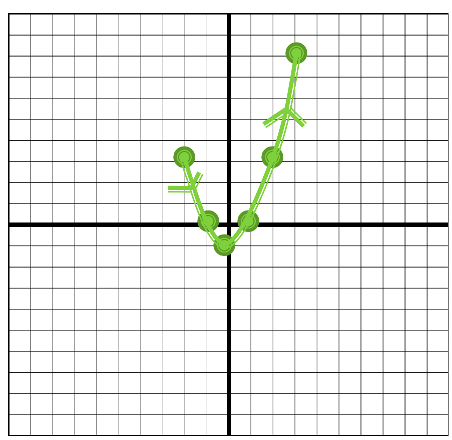
t	x	y
-3	-2	3
-2	-1	0
-1	0	-1
0	1	0
1	2	3
2	3	8
3	4	15



Consider the following parametric relation. Find (x,y) for $-3 \le t \le 2$.

$$x = t + 1$$

$$y = t^2 + 2t$$



Eliminating the Parameter

- In order to **eliminate the parameter** t, choose one of the equations and solve it for t.
- Substitute this expression into the other equation for t.

Eliminate the parameter for the following parametric relation.

$$x = t + 1 \qquad t = x - 1$$

$$y = t^2 + 2t \qquad y = (x-1)^2 + 2(x-1)$$

$$y = x^2 - 1$$

Inverses

- The ordered pair (a, b) is in a relation iff (b, a) is in the **inverse relation**.
- One-to-One Function
  - If f is a one-to-one function with domain D and range R, then the **inverse function** of f, denoted f^{-1}, is the function with domain R and range D.

$$\boxed{f^{-1}(b) = a \ \text{ iff } f(a) = b}$$

  - 1-to-1 functions are those functions whose inverses are also functions

Inverses

- Horizontal Line Test
  - The inverse of a relation is a function iff each **horizontal** line intersects the graph of the original relation in at most one point.
- Inverse Reflection Principle
  - The inverse of a function and the function are symmetric to the line y = x
- Inverse Composition Rule
  - A function f is one-to-one with inverse function g iff

$$\boxed{f\left(g(x)\right) = x \text{ and } g(f(x)) = x}$$

Homework: p.126 #1-31 odd