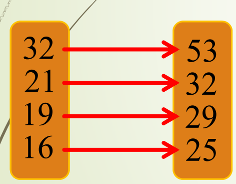
1-2: Functions and Their Properties

Honors Precalculus

Mr. Gallo

Identifying Functions

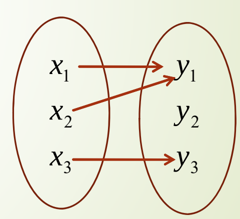
- What is a **Function**?
  - A relation in which each element of the ***domain*** corresponds with ***one and only one*** element in the ***range***.



FUNCTION

$(1,9),(4,-2),(1,-11),(9,10)$

NOT A FUNCTION



FUNCTION

Euhler's Notation: $y = f(x)$

- Notations are used as a short hand
- Used mainly with sentences (equations)
- Letter denotes the name of the function

$$y = 3x - 10$$ Equation

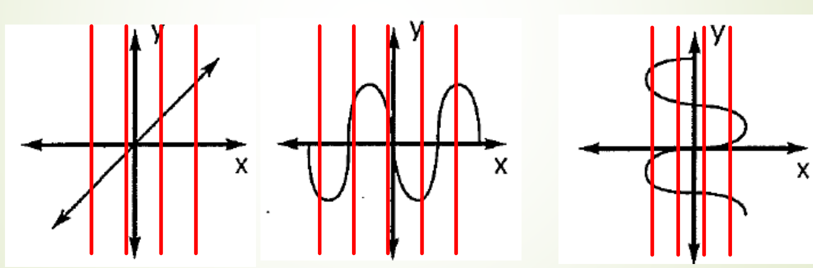
$$f(x) = 3x - 10$$ Function Rule

- **Value of the function**
- Dependent variable
- Output

- **Argument of the function**
- Independent variable
- Input

The Vertical Line Test

- Used with graphs
  - If the vertical line touches the graph in **more than** one point, the graph is not a function.



Domain and Range

Domain

- Set of **inputs**
- Independent Variable

Range

- Set of **outputs**
- Dependent Variable

State the domain of each function using interval notation.

a. $g(x)=\sqrt{4x-1}$

$[\frac{1}{4},\infty)$

$\left\{x \mid x \geq \frac{1}{4}, x \in \mathbb{R}\right\}$

b. $h(t)=\frac{3t^2}{t^2-1}$

$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

$\{t \mid t \neq \pm 1, t \in \mathbb{R}\}$

c. $f(x)=\frac{x-5}{\sqrt{2x-3}}$

$\left(\frac{3}{2},\infty\right)$

$\left\{x \mid x > \frac{3}{2}, x \in \mathbb{R}\right\}$

Finding the Domain

- Implied Domain
  - Defined by the algebraic structure
  - Find where the function does not exist.

Find the domain of $f(x)=\frac{1}{x^2-4}$

$x^2-4=0$

$x=\pm 2$

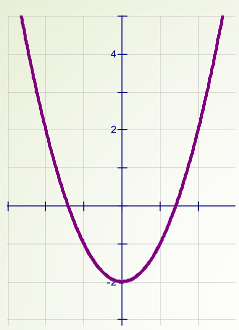
$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

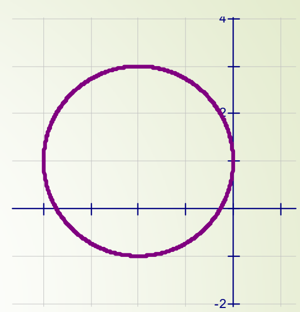
- Relevant Domain
  - Defined by the situation

Finding the Range

- Analytically
  - Look at the equation and determine what values it could be.

    Find the range $f(x) = \frac{2}{x}$
- Graphically
  - Graph and inspect where *y*-values do not exist.
- Algebraically
  - Switch the *x* and *y* variables and solve for *y.* the domain of this expression is the range of the original function. BEWARE the root!!





Domain: $(-\infty, \infty)$

Range: $[-2, \infty)$

Function: Yes

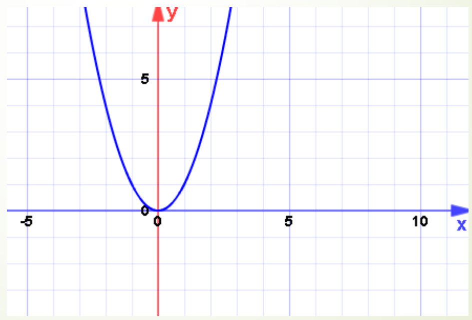
Domain: $[-4, 0]$

Range: $[-1, 3]$

Function: No

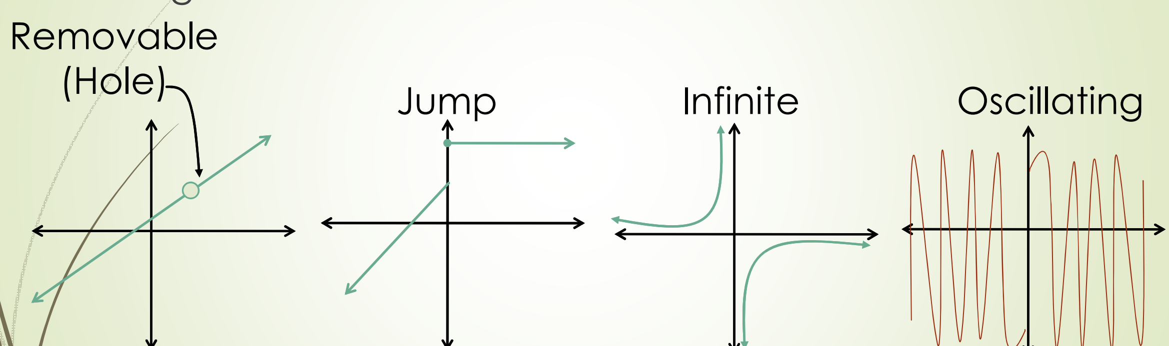
What is a Continuity?

- A function is <u>continuous</u> if you can draw or trace the function without lifting your pencil from the paper.
- For example:



Types of Discontinuities

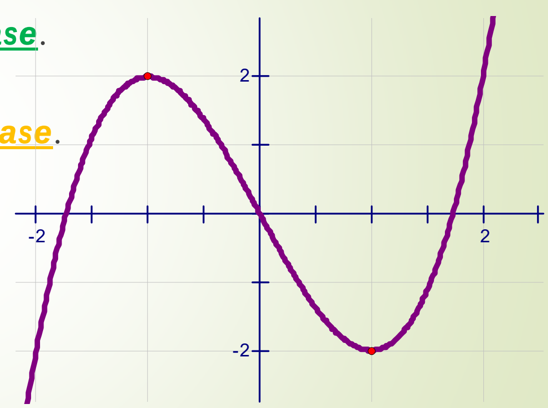
- A function is **<u>discontinuous</u>** if when drawing or tracing it, you must lift your pencil off the paper to continue drawing or tracing.



Homework: p.94 #1-39 odd

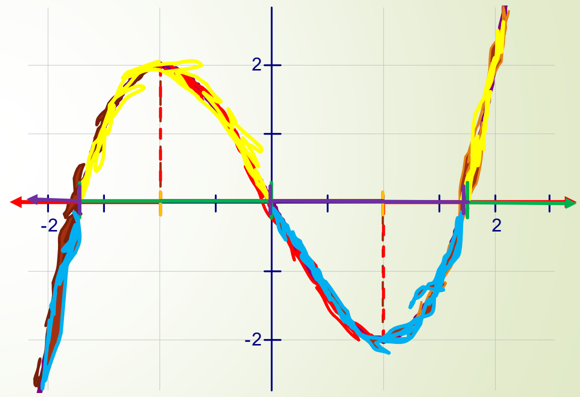
Increasing, Decreasing, Constant, Positive and Negative Behavior

- Behavior of the y values over specific intervals of x values.
  - Increasing
    - As x values ***increase***, y values ***increase***.
  - Decreasing
    - As x values ***increase***, y values ***decrease***.
  - Positive
    - The y values are positive
    - The graph is ***above*** the x-axis
  - Negative
    - The y values are negative
    - The graph is ***below*** the x-axis.



Increasing, Decreasing, Constant, Positive and Negative Behavior

- Identify the intervals the function:
  1. Increases
     $(-\infty,-1)$ and $(1,\infty)$
  2. Decreases
     $(-1,1)$
  3. Is Positive
     $(-1.73,0)$ and $(1.73,\infty)$
  4. Is Negative
     $(-\infty,-1.73)$ and $(0,1.73)$



Increasing, Decreasing, Constant, Positive and Negative Behavior

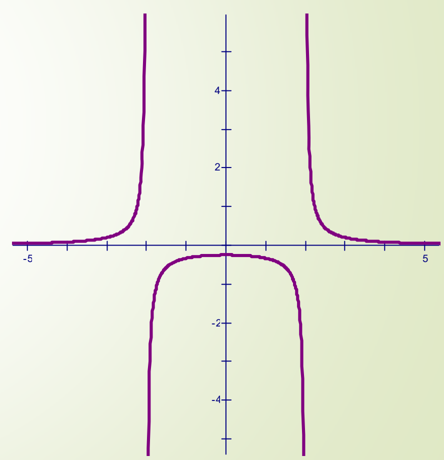
- Constant
  - A function is said to be **constant** on [*a*, *b*] if, for any 2 pts. in [*a*, *b*], a positive change in *x* results in a no change in *y*. i.e., "Slope" is zero.

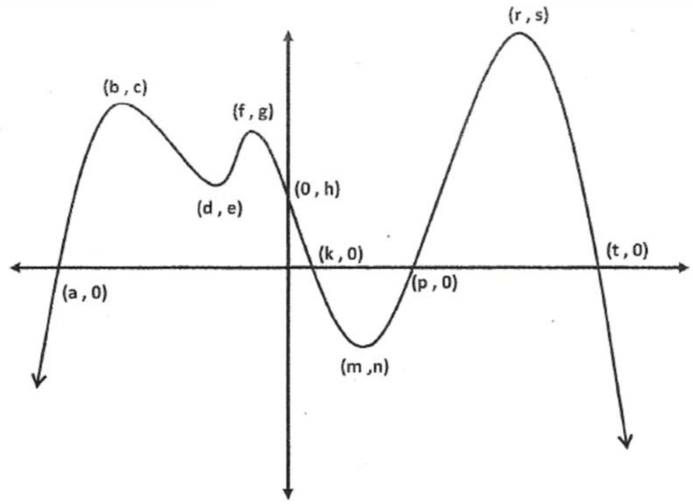
Determine where the following function is inc./dec./const.

$$y=\frac{1}{x^2-4}$$

Increasing: $(-\infty,-2)\cup(-2,0]$

Decreasing: $[0,2)\cup(2,\infty)$





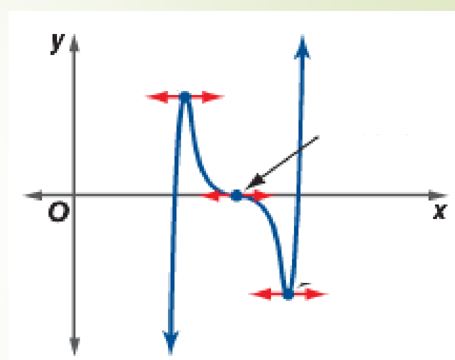
Where is the graph…

Positive: (a,k) (p,t) Negative: $(-\infty,a)(k,p)\ (t,\infty)$

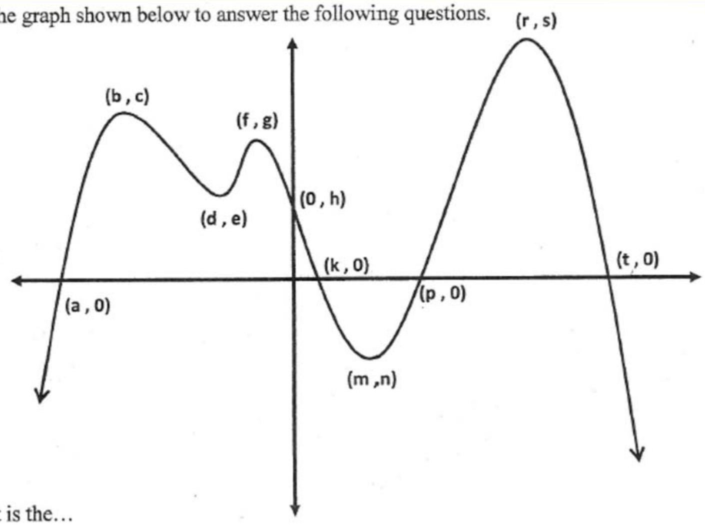
Increasing: $(-\infty,b)(d,f)\ (m,r)$ Decreasing: $(b,d)(f,m)\ (r,\infty)$

Important Points of a Function

- Critical Points
  - Points at which a tangent line to the curve would be horizontal or vertical
- Extrema
  - Point at which function changes its increasing or decreasing behavior
    - Absolute Maximum
    - Relative Maximum
    - Absolute Minimum
    - Relative Minimum
- Point of Inflection
  - Point where graph changes shape
  - Doesn't change increasing or decreasing behavior



Ex. 1: Use the graph shown below to answer the following questions.



What is the…

Absolute max: $y = s$

Absolute min: None

Relative max: $y = c$ $y = g$

Relative min: $y = e$ $y = n$

Boundedness

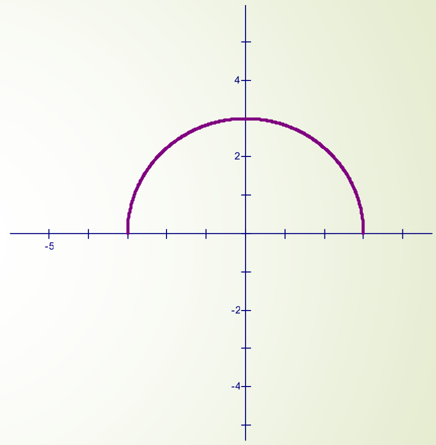
- Bounded Below
  - A function is **bounded below** if there is some number $f(b)$ that is ≤ every number in the range of *f.* $f(b)$ is called the lower bound.
- Bounded Above
  - A function is **bounded above** if there is some number $f(b)$ that is ≥ every number in the range of *f.* $f(b)$ is called the upper bound.
- Bounded
  - A function is **bounded** if it has both an upper bound and a lower bound.

Identify the function as bounded above, below, or both.

$$y = \sqrt{9 - x^2}$$

BDD ABOVE: $y = 3$

BDD BELOW: $y = 0$



Odd and Even Functions

- Symmetry
  - "sameness"; mirror image.
- Even and Odd Functions

Type of Function	Algebraic Test
Functions which are symmetric with respect to the y-axis, are called ***Even Functions***.	For every x in the domain of f, $\boldsymbol{f(-x) = f(x)}$
Functions which are symmetric with respect to the origin, are called ***Odd Functions***.	For every x in the domain of f, $\boldsymbol{f(-x) = -f(x)}$

Prove that the following is an even or odd function or neither.

$$f(x) = x^3 - 2x$$

$$f(x) = x^3 - 2x$$
$$f(-x) = (-x)^3 - 2(-x)$$
$$= -x^3 + 2x$$
$$= -(x^3 - 2x)$$

The answer is the opposite of f(x), therefore the function is odd.

Not every function with the highest exponent being even is an even function.

a. Show by counterexample that f(x) = x4 + x is not an even function.

$$f(1) = 1^4 + 1 = 2$$
$$f(-1) = (-1)^4 + (-1) = 1 - 1 = 0$$

The answers are not the same so the function isn't even.

b. Prove that f(x) = x4 + x is not an even function.

$$f(x) = x^4 + x$$
$$f(-x) = (-x)^4 + (-x)$$
$$= x^4 - x$$

The answers are not the same so the function isn't even.

Homework: p.94 #41-69 odd, 71-76, 78

Analytical Look at Limits

- Given the following function, what happens to f(x) as x gets closer to 3?

$$f(x)=\frac{x^2-5x+6}{x-3}$$

as $x \to 3$, $f(x) \to 1$

- Given the following function, what happens to g(x) as x gets closer to 3?

$$g(x)=x-2$$

as $x \to 3$, $g(x) \to 1$

- Given the following function, what happens to h(x) as x gets closer to 3?

$$h(x)=\begin{cases}\dfrac{x^2-5x+6}{x-3}, & x\neq 3\\ 7, & x=3\end{cases}$$

as $x \to 3$, $h(x) \to 1$

- In general, $\lim_{x\to a} f(x)$ means "as x gets closer to the value a, the values of f are approaching what y value?

Limit Notation

- Two-sided Notation: $\lim_{x \to a} f(x) = L$
  - Read as "the limit of *f*(*x*) as *x* approaches ***a*** is *L*."

- One-sided Notation:

$$\lim_{x \to a^+} f(x) = L$$

  - Read as "the limit of *f*(*x*) as *x* approaches ***a*** from the right is *L*."

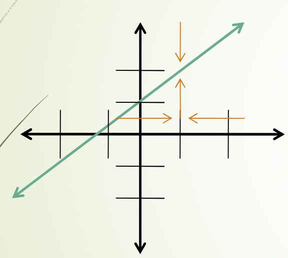
$$\lim_{x \to a^-} f(x) = L$$

  - Read as "the limit of *f*(*x*) as *x* approaches ***a*** from the left is *L*."

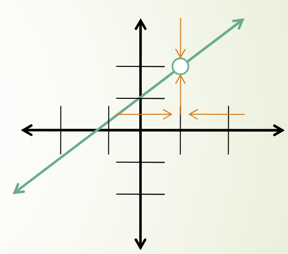
Graphical Evaluation of Limits

- Evaluate the following limits graphically:

1.) $\lim_{x \to 1} x+1 = 2$



2.) $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$

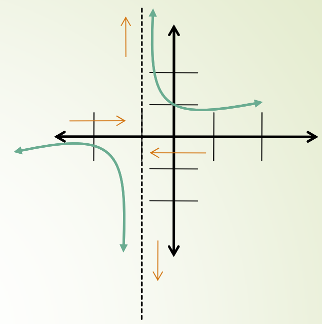


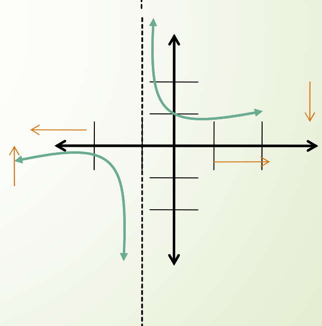
3.) $\lim_{x\to -1^+} \frac{1}{x+1} = +\infty$

4.) $\lim_{x\to -1^-} \frac{1}{x+1} = -\infty$

5.) $\lim_{x\to -\infty} \frac{1}{x+1} = 0$

6.) $\lim_{x\to \infty} \frac{1}{x+1} = 0$





Asymptotes

- Vertical Asymptotes
  - The line $x = a$ is a vertical asymptote of the graph of $f(x)$ if

$$\lim_{x\to a^-} f(x) = \pm\infty \text{ or } \lim_{x\to a^+} f(x) = \pm\infty$$

  - Factor and set the denominator equal to zero. Then describe the behavior at each asymptote!
- Horizontal Asymptotes
  - The line $y = b$ is a horizontal asymptote of the graph of $f(x)$ if

$$\lim_{x\to \pm\infty} f(x) = b$$

  - Take the limit as x approaches $\pm\infty$. Algebraically, usually we divide each term by the highest power of x.

Find the vertical asymptotes in each of the following and determine the behavior at each asymptote.

1.) $f(x)=\frac{x^2-4}{x-2}$

$\frac{(x+2)(x-2)}{x-2}=x+2$

No Vertical Asymptotes!!!

2.) $f(x)=\frac{12x+2}{x^2-1}$

$\frac{2(6x+1)}{(x-1)(x+1)}$

$(x-1)(x+1)=0$

Vertical Asymptotes
x = ±1

Behavior

$\lim_{x\to-1^+}=\frac{12x+2}{x^2-1}=\frac{-10}{0^-}=\infty$

$\lim_{x\to-1^-}=\frac{12x+2}{x^2-1}=\frac{-10}{0^+}=-\infty$

$\lim_{x\to1^+}=\frac{12x+2}{x^2-1}=\frac{14}{0^+}=\infty$

$\lim_{x\to1^-}=\frac{12x+2}{x^2-1}=\frac{14}{0^-}=-\infty$

Find the horizontal asymptote(s) in each of the following:

1.) $f(x)=\frac{3x^3-4}{2x^2-5x^3}$

$$\lim_{x\to\pm\infty}\frac{\frac{3x^3}{x^3}-\frac{4}{x^3}}{\frac{2x^2}{x^3}-\frac{5x^3}{x^3}}=\lim_{x\to\pm\infty}\frac{3-\frac{4}{x^3}}{\frac{2}{x}-5}=\lim_{x\to\pm\infty}\frac{3-0}{0-5}=-\frac{3}{5}\qquad y=-\frac{3}{5}$$

2.) $f(x)=\frac{x}{x^2-1}$

$$\lim_{x\to\pm\infty}\frac{\frac{x}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}=\lim_{x\to\pm\infty}\frac{\frac{1}{x^2}}{1-\frac{1}{x^2}}=\lim_{x\to\pm\infty}\frac{0}{1-0}=0\qquad y=0$$

End Behavior

$$\frac{Term\ w/Greatest\ Power\ (\mathrm{N})}{Term\ w/Greatest\ Power\ (D)} = \text{End Behavior}$$

- End behavior describes how the graph behaves at its "outer limits". Determine the limit as $x \to \pm\infty$.
- Can find end behavior by:

$$f(x) = -x^4 + 8x^3 + 3x^2 + 6x - 80$$

$$\lim_{x\to\infty} f(x) = -\infty$$

- End behavior will mimic the greatest power $-x^4$ so:

$$\lim_{x\to-\infty} f(x) = -\infty$$

$$f(x) = \frac{x}{x^2 - 2x + 8}$$

$$\lim_{x\to\infty} f(x) = 0$$

- When x is in the denominator, the limit will always go to 0. $\frac{Greatest\ Power\ (\mathrm{N})}{Greatest\ Power\ (D)} = \frac{x}{x^2} = \frac{1}{x}$

$$\lim_{x\to-\infty} f(x) = 0$$

Possible End Behaviors

- There are three possible end behaviors:

1. The values of *f(x)* can increase or decrease without bound (to ∞ or −∞)
2. The values of *f(x)* can approach some number *L*
3. The values of *f(x)* can follow neither of these patterns (such as oscillating between two values).

Homework: WS 1-2 Limits